Manipulative Advertising by a Monopolist*

[PRELIMINARY DRAFT]

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Abstract

We analyze an information manipulation game where consumers estimate from a noisy and possibly biased signal the unobserved quality of the product that a monopolist produces. The bias is the result of the costly and hidden advertisement strategy of the monopolist. We show that the monopolist of all quality types engage in manipulative advertising. The intensity of manipulative advertising does not necessarily increase with quality. The firm type with the highest manipulation is always able to increase its demand and all firm types can succeed in effective manipulation. Manipulative advertising is not necessarily harmful for consumers as it may improve consumer surplus by increasing the consumption of higher quality products and lowering the lower quality ones for relatively high prices. To maximize the beneficial effects of manipulative advertising a regulator may require a fixed fee for advertising from the monopolist. We finally show that endogenous and informative pricing strategies by the monopolist do not substitute costly advertising but complement it.

JEL: D8, K4, L1, L4.

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1 Introduction

People are constantly exposed to messages by politicians and firms that are manipulative by design. When the interests of senders and receivers are not fully aligned, these messages may harm the receivers to the extent that they are persuasive. Even though voters and consumers are typically aware of such conflicts of interest and do not take senders' messages at face value, oftentimes, they end up voting for the wrong candidate or making a purchase that is ex-post inferior. Such failures in information aggregation could be attributed to some characteristics of signal receivers that prevent them from filtering possible biases from the information they receive. Indeed, studies propose bounded rationality (e.g. Mullainathan et al. 2008; Bertrand et al. 2010) or biased prior beliefs (Gentzkow and Shapiro, 2006) on the part of signal receivers as potential explanatory factors. In this paper, we offer an alternative explanation that focuses on the informational frictions in the market that generates aggregate bias in consumers' posterior beliefs.

We analyze an information manipulation game where a monopolist engages in costly and manipulative advertising of an experience good (Nelson, 1970) it produces. In our model, the consumers are fully rational, immune from behavioral biases, and hold beliefs that are a priori unbiased. Persuasive power of ads stems from an informational problem buyers face. Since consumers cannot verify the quality of the experience good before purchase, they draw on various sources of information such as word-of-mouth advice or online reviews to make a decision. Yet, consumers differ with respect to the sources of information they are exposed to. The social network one belongs to, the type of media one follows and many other factors can affect one's information sampling process in ways that are difficult to predict (due to an idiosyncratic component) and that vary across different people. In other words, the opinions or reviews consumers hear or read about a product can be thought as containing random biases of different strength and direction that may not be perfectly anticipated even by consumers themselves. The goal of this paper is to explore the implications of such an information collection process. The main question we tackle is whether firms can exploit this communication environment through manipulative ads.[1] Our answer is

[1] Empirical evidence suggests that in some sectors we indeed observe manipulative practices that are reminiscent of the mechanism we propose. For example, Mayzlin, Dover and Chevalier (2014) show that the evidence from online hotel reviews are consistent with the employment of fake promotional reviewers by hotels. Another example is pharmaceutical research, where pharmaceutical companies have incentives to manipulate the research process to get more favorable research outcomes (see e.g.

affirmative. When advertising is not directly observable and arrives with some noise, firms can indeed tailor their ads so as to generate an inference problem for buyers, even at the aggregate level.

In our model, a monopolist tries to sell an experience good of either high or low quality to a continuum of consumers with unit demand. We start with the assumption that both monopoly types charge the same price (which is allowed to vary exogenously) and relax this assumption later on.[2] Product quality is directly observable to the monopolist but not to consumers. Instead, consumers receive a noisy signal about it. Manipulative advertising is modeled as an unobserved action by the monopolist that shifts the mean of this otherwise unbiased signal. Under this communication environment, although consumers perfectly anticipate the advertising strategies of each type of monopolist and update their beliefs accordingly, in equilibrium, the monopolist is able to shape average beliefs via manipulation and consequently influence demand for its product. The aggregate bias in consumer beliefs stems from the fact that private signals contain a random noise, and ad levels are unobserved and state-dependent (differ by the type of monopolist). As a result, consumers not only remain uncertain about the state, but they also assign a different likelihood to each state than they would in the absence of manipulation. We demonstrate that this manipulation technology always helps the monopoly type with the highest level of advertising in increasing its sales. Specifically, the low [high] quality monopolist will enjoy higher demand under manipulative advertising whenever prices are lower [higher] than the quality level consumers expect under their common prior beliefs. As a result, the welfare consequences of manipulation depend crucially on the price level. In particular, the effect on ex-ante consumer surplus tends to be negative at low prices and positive at high prices. Firm profits, on the other hand, depend not only on sales but also costs of ads. We show that sellers' ability to successfully manipulate consumers inevitably leads to some wasteful spending on ads. The two types of the monopolist engage in an implicit arms race where false ads by one type trigger more effort by the other to mislead consumers. We argue that the problem of designing an optimal advertising

Finucane and Boult 2004 and Sismondo 2008).

[2]In the benchmark model, we assume that the price of the product is exogenous and can take any value between the levels of value that the product can take. In Section 6 we allow the monopolist to choose price strategically along with the advertisment levels. We demonstrate that the range of exogenous prices we consider in the benchmark model can be supported by pooling equilibria, where the price is not informative. When we allow for mixed pricing strategies by the monopolist, we also find that there exist partially separating equilibria in which the price level can provide some partial information about the quality.

policy under such an environment is not trivial since it requires identification of market specific details.

While most of the theoretical literature on advertising has focused on truthful advertising, there are some recent models of false advertising to which our paper relates. The existing work modeled false advertising in various ways. In some of these studies, false messages are taken as given, instead of being derived as an equilibrium choice of firms, and consumers are assumed to take these messages at face value rather than rationally discounting them (e.g. Hattori and Higashida 2012). Some papers allow for false or unsubstantiated claims and study how various regulations on product advertising influence the degree of information acquisition and disclosure by firms. Yet they take the strength of claims about product quality as exogenously given (e.g. Corts 2013). False claims are supported in equilibrium only when firms are uncertain of their own product quality, i.e. there is no intentional misinformation by firms (e.g. Corts 2014). Some recent papers feature intentionally false claims (Rhodes and Wilson 2015, Piccolo, Tedeshi and Ursino 2015, 2016). Some equilibria in these models feature deceptive advertising by low quality firm, which does affect consumers' posterior beliefs about product quality. While these papers let the low-quality firm pretend to be a high-quality firm, they do not allow the high type to respond with counter ads (i.e. exaggerate its product quality). These models make the restrictive prediction that the only firms which engage in costly advertising are the low quality ones. In contrast, the manipulation framework we have in this paper allows for manipulative advertising by all types of the firm. Our project is not the only study to model ads as an unobservable bias sellers introduce to otherwise unbiased but noisy signals. Drugov and Troya-Martinez (2015) adopt a similar signal jamming approach. In their paper consumer tastes are heterogeneous, and the quality of the match between the firm's product and an individual consumer's taste is observable neither by the consumer nor the firm. The paper features positive bias in seller's advice (or ad) in equilibrium. However, since the seller cannot condition this bias on the quality of a particular match, false advice does not affect total sales. In this project, we have a different motivation; namely to support an equilibrium in which false advertising has an effect on market demand as well as individual beliefs. In contrast to Drugov and Troya-Martinez (2015), our paper features asymmetric information between firms and consumers, such that firms observe the quality of their product while consumers remain uncertain about it.

Information manipulation by hidden unobservable actions have been studied in various other contexts. Earlier work such as by Matthews and Mirman (1983), and Fudenberg and Tirole (1986) focused on manipulation of unobservable pricing decision of an incumbent firm to deter a potential entrant (For a more recent work along this line see Mirman et. al , 2014). More recently, Edmond (2013), Caselli, Cunningham, Morelli and Moreno Barreda (2014), and Aköz and Arbatlı (2016) studied information manipulation in political context. We contribute to this literature by analyzing the impact of the additional variation (in our case this is price) in the benefit function of the manipulating player. Price has three types of effects on manipulation. On the one hand, higher prices raise the marginal benefit of manipulation, increasing the incentive for advertising for both types of the monopolist. On the other hand, higher prices reduce consumer surplus, thereby lowering the mass of buyers. Because of this secondary effect, we find that for sufficiently high prices, manipulation is close to zero for both types of the monopolist. The third effect of price is to determine the relative advantage in the implicit competition between the two types of the monopolist. We find that lower [higher] prices enable the low[high]-quality type to manipulate a larger set of consumers.

Our information manipulation approach is related to the models of Bayesian persuasion in the sense that the monopolist manipulates the information structure to persuade Bayesian consumers. However, in our model the monopolist learns its type before committing to a manipulation plan and the cost of persuasion increases with the average bias in the information receivers have. This is in contrast with the canonical model of Bayesian persuasion by Kamenica and Gentzkow (2011), and with the follow up study in Kamenica and Gentzkow (2014) where, rather than manipulation, providing more precise information is costly. We show in Section 5 that with such a model that gives commitment power to the monopolist, there would be no manipulative advertising if it does not increase the ex-ante profit level. Therefore, there would always one type of monopoly which would end up engaging in no manipulation. In our model, on the other hand, manipulation by each type of the monopolist increases the incentive to do more manipulation by the other type. Therefore, we find that both types of the monopolist engage in manipulation even if consumers perfectly adjust their posterior beliefs, thereby canceling any effect of manipulation on revenues. This type of implicit competition between various types of the persuader is absent in Kamenica and Gentzkow (2011) and Kamenica and Gentzkow (2014).

In Section 6 we relax the assumption that the price is uninformative about the quality. When we allow for mixed pricing strategies by the monopolist, we find a set of partially separating equilibria, in which the low quality monopolist randomizes between the minimum price and a high price, while the high quality monopolist always chooses the same high price. Therefore, when consumers observe the high price, they assign a higher likelihood to the monopolist being a high type than what the prior belief assigns. This type of randomized strategies by the low-quality type can also be found in signaling models of advertising by Janssen and Roy (2010) and Rhodes and Wilson (2015). However, in our model consumers still consult to their noisy signal for making the purchasing decision, since the price is not fully informative. Thus, the partially separating equilibria combines randomized signaling strategies by firms and the information manipulation approach that we have in this paper. We find, consistently with the main finding in the benchmark model, that relatively higher prices are associated with higher advertising by the high quality type monopolist. On this path of equilibria higher prices are more informative since the low quality monopolist adopts lower mixing probability of not choosing the minimum price. This implies that when the price is higher the monopolist can influence more consumers by advertising, which also increases the level of manipulative advertising. Therefore, the information that price carries is complement, not substitute, for the manipulative advertising by the monopolist.

The rest of the paper is organized as follows. Section 2 lays out the model and characterizes the equilibrium. Section 3 presents the main analytical and numerical results regarding the effect of manipulation on total demand and consumer surplus. It also provides intuition about why and when manipulative ads can become effective in influencing average behavior. Section 4 discusses the policy implications. Section 5 discusses two extensions of the model. We first analyze the different implications of assuming that the monopoly has commitment power. Second, we discuss the effect of signal precision on manipulation. Section 6 endogenizes the price. Finally section 7 concludes the paper.

2 Model

Consider a monopolist releasing a new product whose quality is unknown to consumers $i \in [0,1]$. The monopolist can be one of two types $j \in J = \{L, H\}$ based on the quality of the product it supplies. In particular, each type produces a product of quality v_j such that $0 \leq v_L < v_H$. We index the monopolist's type by the product quality it supplies. The marginal costs of production do not depend on product quality and are normalized to zero without loss of generality.[3]

Consumers do not know the product quality but receive a private signal x_i. The monopolist of type j can uniformly shift the mean of the signal by engaging in costly manipulative advertisement. We assume that the noisy signal x_i is separable to three components: the true quality v_j of the product, the manipulative action by the monopolist, and the random noise. In particular,

$$x_i = v_j + a_j + \varepsilon_i,$$

where $a_j \equiv a(v_j) \geq 0$ is the degree of manipulative advertisement chosen by monopolist j and $\varepsilon_i \sim F$ is an idiosyncratic random noise in the advertising signal with a known cumulative distribution function F and a corresponding density function f.[4] Prior to observing their private advertising signals, consumers hold a common prior belief $G : \{v_L, v_H\} \rightarrow [0, 1]$ about the product quality (hence the type) of the monopolist they face. Following Assumption 1 states the restrictions we place on the noise distribution and parametrizes the prior distribution.

Assumption 1 *Density function f for the random noise in advertising signals is continuous, log-concave, symmetric around zero and unimodal with unbounded support and finite moments. G is a discrete distribution function such that $Pr(v_L) = g \in (0, 1)$ and $Pr(v_H) = 1 - g$.*

Upon observing her private signal each consumer decides whether to buy one unit of the monopolist's good. Ex-post utility of each consumer who purchased the product

[3]Our main results can be generalized after an appropriate normalization to the case where high quality monopolist has a higher marginal cost of production.

[4]Assuming idiosyncratic noise generates an informational heterogeneity among consumers. If we assume that the random noise is common so that all consumers receive the same signal, the market demand, in terms of share of consumers, would be either 0 or 1. From the firm's perspective, each manipulative action would correspond to a different distribution of signals that consumers may receive, therefore a different probability of full demand. All of our results with this probabilistic interpretation of demand would carry out in this set up as well.

An alternative way to introduce heterogeneity among consumers is to assume that preferences differ among consumers. Specifically, one can assume that consumer i receives the following payoff $v_L + \psi_i - p$,when she purchases the good at price p. Here ψ_i is the individual match quality between the consumer and the product, where ψ_i is distributed with c.d.f. $H(\cdot)$ and support (v_L, v_H). This specification would generate a downward sloping demand curve for every signal public signal x.

offered by monopolist j is given by

$$u = v_j - p_j \tag{1}$$

where p_j is the unit price announced by monopolist j and v_j is the product quality. If the product is not purchased, then $u = 0$. We denote the binary purchase decision of the consumer by a function $s(x_i) \in \{0, 1\}$ so that i buys the good, i.e., $s(x_i) = 1$ if and only if $E[v_j|x_i] \geq p_j$.

We assume for the benchmark model that the market has a regulated price $\bar{p} \in (v_L, v_H)$ and the monopoly commits to setting the price at $\bar{p}$ prior to learning its product quality type. The regulation on the price, which could be exogenously regulated by a public authority or be the outcome of the decision of the monopoly before supplying to this market, strips the price off its informative function and reduces it to a parameter of transfer of surplus from consumers to the monopoly.[5] We allow for endogenous and informative pricing in Section 6.

Since the noisy signals are the only information sources for consumers, they base their decision solely on the information they receive. Therefore, the sales of monopolist of type j is given by

$$S(v_j, a_j) = \int_0^1 s(x_i)di = \int_0^1 s(v_j + a_j + \varepsilon_i)di, \tag{2}$$

where $s(x_i)$ is indicator function for the purchasing decision made by the consumer.

The profit to the monopolist of type j is given by

$$\pi_j = p_j S(v_j, a_j) - C(a_j), \tag{3}$$

where $C(.)$ is the cost of advertising and it is equal to

[5]If the monopoly could choose the price strategically, the price would not necessarily have any informative value. Indeed, it is possible to support every exogenous price as an outcome of a pooling equilibrium if the value of the low quality v_L equals 0. However, endogenizing price has non-trivial implications for the characterization of the set of equilibria. Since we focus on manipulative advertising in this section, we abstract away from any informative value of pricing by assuming that it is exogenously fixed.

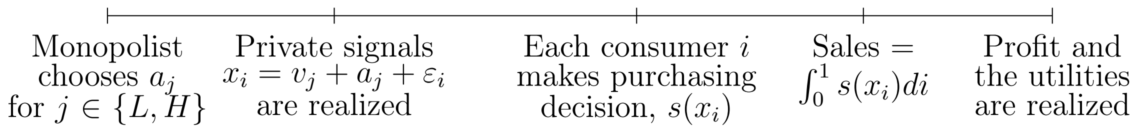

Figure 1: Timeline

$$C(a_j) = \begin{cases} 0 & \text{if} \quad a_j = 0 \\ \bar{c} + c(a_j) & \text{if} \quad a_j > 0, \end{cases} \tag{4}$$

where $\bar{c} \geq 0$ is the fixed cost of manipulative advertising and $c(.)$ is the variable cost of manipulative advertising. We assume for the benchmark model that the fixed cost of advertising to be 0. We discuss the implications of corner solutions caused by non-zero fixed cost of advertising in Section 4.

Following Assumptions 2 and 3 state the restrictions we put on the advertising costs. Assumption 2 guarantees rules out the trivial cases, where the monopolist does not prefer to do any advertising. Assumption 3 imposes strict concavity to the profit function of the monopoly and therefore a unique advertisement level at each quality level.

Assumption 2 *The cost function $C(.)$ satisfies $C'(0) = 0$, and $C''(a)$, $C'(a) > 0$ for all $a > 0$.*

Assumption 3 $\min_{a \geq 0} C''(a) > p \max_x f'(x)$.

The timeline of the game is illustrated in Figure 1

We employ symmetric pure-strategy Perfect Bayesian Equilibrium (PBE) as the solution concept for our analysis. Intuitively, PBE requires sequential rationality and Bayesian update for posterior beliefs whenever possible. More formally a strategy profile is the advertisement choice of each type of monopoly (a_L, a_H) and the purchasing decision of the consumer after observing the noisy signal x $s(x)$. Then a strategy profile

$$\langle a_L^*, a_H^*, s^*(\cdot) \rangle$$

accompanied with the posterior belief of a consumer $\mu(x)$ who observed the signal x is a symmetric pure-strategy PBE if and only if

$$a_L^* \in \underset{a_L \in \mathbb{R}^+}{\operatorname{argmax}} \bar{p} S(a_L, a_H^*, v_L) - C(a_L)$$
$$a_H^* \in \underset{a_H \in \mathbb{R}^2}{\operatorname{argmax}} \bar{p} S(a_L^*, a_H, v_H) - C(a_H)$$
$$S(a_L^*, a*_H, v_j) = \int_{-\infty}^{\infty} s(x) f(x - v_j - a_j^*) dx$$
$$s(x) = \begin{cases} 1 & \text{if} \quad \sum_j (v_j - \bar{p}) \mu(x)(v_j) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$
$$\mu(x)(v_j) = \frac{Pr(x|v = v_j) G(v_j)}{Pr(x|v = v_L) g + Pr(x|v = v_H)(1 - g)}, \tag{5}$$

where $Pr(x, p|v = v_j)$ is the probability that the signal x and the price p to be realized on the equilibrium path given that the type of the monopoly is v_j.

2.1 Equilibrium Analysis

Assumption 1 puts some regularity conditions on the posterior beliefs that consumers could have. In particular, consumers' posterior expectation of quality given the signal they receive is strictly increasing with the signal. Therefore, the purchasing decision of consumers admits a simple monotonic threshold structure. To show that we will first suppose such a monotonic strategy by consumers and calculate the advertising decision of the monopoly, and then we will confirm that consumers' decisions confirm our supposition.

Suppose that the consumers follow a monotonic threshold strategy $\bar{x}$ such that $s(x_i) = 1$ if and only if $x_i \geq \bar{x}$. Then the monopolist chooses the level of manipulative advertisement a_j that solves

$$\max_{a_j \geq 0} \quad \bar{p}[1 - F(\bar{x} - v_j - a_j)] - C(a_j) \quad \text{for all } j \in J \tag{6}$$

The first order condition to this problem is given by

$$\bar{p} f(\bar{x} - v_j - a_j) = C'(a_j) \quad \text{for all } j \in J \tag{7}$$

Assumption 1 combined with Assumption 2 ensures that for any j an interior

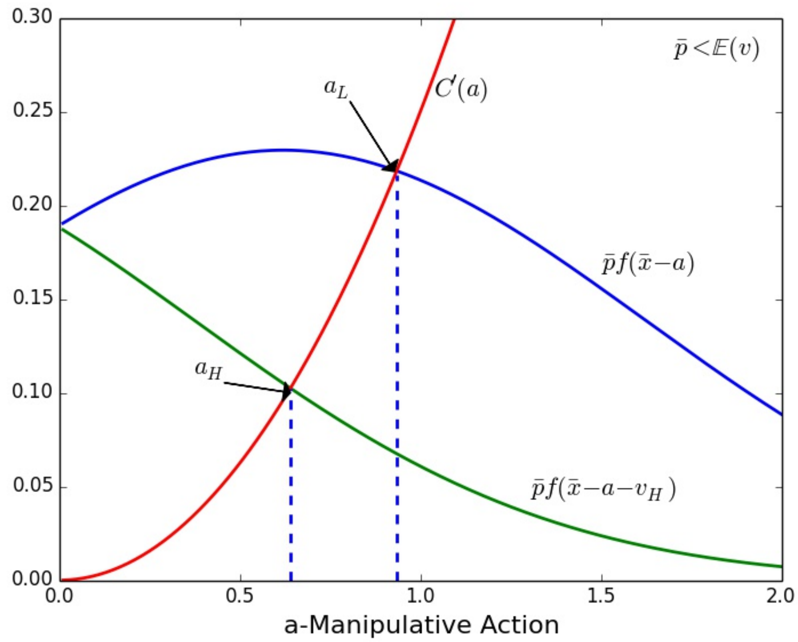

Figure 2: Equilibrium Values of a_L and a_H

solution $a_j > 0$ to this problem exists. Moreover, if we further assume Assumption 3, we can ensure that the solution is also unique. Figure 2 illustrates the determination of the manipulative actions a_L and a_H for a price $\bar{p}$ lower than the prior expected product quality $\bar{p} < \mathbb{E}(v)$.

Given the levels of manipulative advertising, consumers form their equilibrium beliefs using Bayesian update as follows. When a consumer receives an advertising signal x, her posterior expectation of the product quality will be

$$\mathbb{E}(v|x) = \frac{\sum_j v_j f(x - v_j - a_j)G(v_j)}{\sum_j f(x - v_j - a_j)G(v_j)} \tag{8}$$

Therefore, at price $\bar{p}$, a consumer will be indifferent between buying and not buying the good if and only if she receives a signal $\bar{x}$ which satisfies

$$\begin{aligned} \mathbb{E}(v|\bar{x}) = \frac{\sum_j v_j f(\bar{x} - v_j - a_j) G(v_j)}{\sum_j f(\bar{x} - v_j - a_j) G(v_j)} &= \bar{p} \Leftrightarrow \\ \sum_j (v_j - \bar{p}) f(\bar{x} - v_j - a_j) G(v_j) &= 0. \end{aligned} \tag{9}$$

Four equations given by (7) and (9) determine the equilibrium with manipulative advertising.

If there were no manipulative advertising, then consumers would form their posterior beliefs as

$$\mathbb{E}(v|x) = \frac{\sum_j v_j f(x - v_j) G(v_j)}{\sum_j f(x - v_j) G(v_j)},$$

where x is any signal that a consumer might receive. Then, given a price $\bar{p}$, the condition that determines the signal $\underline{x}$ under which the consumer is indifferent between buying and not buying is given by

$$\begin{aligned} \mathbb{E}(v|\underline{x}) = \frac{\sum_j v_j f(\underline{x} - v_j) G(v_j)}{\sum_j f(\underline{x} - v_j)} &= \bar{p} \Leftrightarrow \\ \sum_j (v_j - \bar{p}) f(\underline{x} - v_j) G(v_j) &= 0. \end{aligned} \tag{10}$$

The consumers' problem of estimating the quality level is not trivial. A Bayesian consumer knows that she is manipulated by the Monopoly, thus she has to adjust her posterior belief accordingly. By Assumption 1, if there were no manipulation in the information that the consumer receives, a higher signal would directly translate into a higher likelihood of the H-type Monopoly. However, when there is manipulation, a higher signal might mean a higher quality or higher manipulative advertisement by the Monopoly. For the posterior expectations to be monotonic in signals, the quality difference should dominate the difference in manipulation, which requires a sufficient increase in cost of advertisement compared to the response of consumers to higher signals. This way, the same noise value ε would lead to a higher signal x when the underlying quality is high. That is $v_H + a_H + \varepsilon > v_L + a_L + \varepsilon$. Following Lemma1 shows that Assumption 3 or the weaker Assumptionas:monovav below are sufficient for that.

Assumption 4 *Suppose that cost function and the quality levels satisfy the following inequality*

$$C'(v_H - v_L) \geq v_H f(0).$$

Lemma 1 *Suppose that Assumptions 1 and 2 hold, and that consumers use a unique finite signaling threshold* $\bar{x}$. *Assumption 3 or Assumption 4 implies that* $a_H + v_H > a_L + v_L$.

We present all omitted proofs in the Appendix A.

Note that Assumption 4 is a weaker condition than Assumption 3, since simple integration shows that Assumption 3 implies Assumption 4. Therefore, for thefollowing Lemma, which states that the posterior expectation is monotonic insignal, to hold we do not require a strictly concave profit function but only a sufficient quality difference between two types of Monopoly.

Lemma 2 *Suppose that Assumptions 1 and 2 and Assumption 4 hold and that* $a_L + v_L < a_H + v_H$. *Then,* $E[v|x]$ *is strictly increasing in* x.

The most immediate consequence of Lemma 2 is the existence and uniqueness of a threshold $\bar{x}$ for every manipulation pair (a_L, a_H) by the Monopoly.

Corollary 1 *Suppose that Assumptions 1, 2 and 4 hold and that* $a_L + v_L < a_H + v_H$. *For any price* $\bar{p} \in (v_L,\ v_H)$, *there is a unique threshold* $\bar{x}$ *for advertising signals such that only those consumers with* $x > \bar{x}$ *will purchase the product. It is defined by the condition* $E[v|\bar{x}] = \bar{p}$.

Combining the first order conditions (7) and Corollary 1 establishes the existence of an equilibrium with positive manipulative advertisement levels conditioned on a price, $\bar{p}$. Following Theorem provides conditions for existence and uniqueness of such equilibria.

Theorem 1 *Suppose that Assumptions 1, 2 and 4 hold. For each price level* $\bar{p} \in (v_L, v_H)$, *there exists a Perfect Bayesian Equilibrium, characterized by manipulative advertisement levels* a_L, $a_H > 0$, *and a signaling threshold* $\bar{x}$, *such that any consumer* i *who receives signal* x_i *purchases the product if and only if* $x_i \geq \bar{x}$.

Moreover, if Assumption 3 holds as well, for price level $\bar{p} \in (v_L, v_H)$, *there exists a unique equilibrium.*

One of the important implications of Theorem 1 is that there is always some advertising. The Monopoly chooses to spend some of its resources on information manipulation whether it produces a low quality product or a high quality one. This is one of the advantages of the information manipulation approach that we take in this model. This is in contrast with informative advertisement models and most of the variants of false advertisement models, where usually only one of the quality types engage in advertising (see Rhodes and Wilson , 2015 for example).

3 The Implicit Competition Between Types and Effective Manipulation

Theorem 1 implies that both types of the Monopoly do manipulative advertising. That is, the Monopoly spends some of its resources for manipulative advertising whether it is of high or low quality. Actually, it is not optimal from an ex-ante perspective for the Monopoly to do advertising irrespective of its quality level. To see this, suppose that a regulator publicly announces to reduce the manipulative advertising by the Monopoly uniformly until the advertisement level by at least one type of Monopoly vanishes. That is, if the expected advertisement levels of the Monopoly are such that $a_L > a_H > 0$, the agency reduces the advertisement level by a_H, so that L-type's advertisement level is $a_L - a_H$ and H-type's advertisement level is 0.

Such an adjustment in the advertisement levels leads consumers to uniformly shift their posterior estimation of the quality since each signal contains less manipulation. Therefore, the threshold that the consumers employ also reduces to adjust for the uniform reduction in the manipulation levels. In particular, the posterior expectation would be

$$\frac{\sum_j v_j f(\bar{x} - \min\{a_j\} - v_j - (a_j - \min\{a_j\}))G(v_j)}{\sum_j f(\bar{x} - \min\{a_j\} - v_j - (a_j - \min\{a_j\}))G(v_j)}$$
$$= \frac{\sum_j v_j f(\bar{x} - v_j - a_j)G(v_j)}{\sum_j f(\bar{x} - v_j - a_j)G(v_j)} = \bar{p}.$$

That is, the Monopoly achieves the same amount of sales by spending uniformly less on advertising, which makes the Monopoly better off. Then, why does the Monopoly engages in such an excessive level of advertising?

One of the factors that lead the Monopoly to increase its advertisement level even further is the implicit competition between the two types of Monopoly. When the Monopoly observes its quality level, it takes the advertisement level by the other type of Monopoly as given. If the consumers expect that the other type of Monopoly would engage in more aggressive manipulation, then the Monopoly would increase its manipulation level as well to balance the consumers' lower level of estimated quality level. In other words, each type of the Monopoly behaves as if it is in an "arms race" with the other type of Monopoly. The more aggressive a type is in its advertisement decision, the more advantageous it is since consumers cannot make a type specific adjustment in their expectations but an average one.

This implicit arms race between the two types of Monopoly gives the price a second function apart from its direct effect on revenues: determining the competitive advantage of each type against the other type of Monopoly. We show below that the low (high) quality type is more advantageous for low (high) prices. Intuitively, lower prices are associated with the lower risk of negative consumer surplus $\bar{p} - v_L$. In particular, when the price is lower than the prior expected quality level, the monopoly expects that majority of consumers will receive a favorable enough signal and decide to purchase the product, even if the quality is low and both types engages in the same level of manipulation. In such cases, the low quality type has more power to shift the expectations of consumers by manipulation. To see this consider the following modification of equation (9)

$$(v_H - \bar{p})(1-g)f(\bar{x} - a_H - v_H) = (\bar{p} - v_L)gf(\bar{x} - a_L - v_L). \tag{11}$$

When $(\bar{p} - v_L)g < (v_H - \bar{p})(1-g)$ or equivalently

$$\bar{p} < (1-g)v_H + gv_L,$$

the threshold signal $\bar{x}$ that makes consumers indifferent between purchasing and not purchasing the product is such that

$$f(\bar{x} - a_H - v_H) < f(\bar{x} - a_L - v_L),$$

which immediately implies by first order conditions (7) that $a_L > a_H$. Indeed, consumer indifference condition (11) gives us a complete characterization of the comparison of the manipulation levels in terms of price levels.

Proposition 1 *The manipulation levels* $a_H = a_L$ *if and only if* $\bar{p} = (1-g)v_H + gv_L$. $a_H > a_L$ *if and only if* $\bar{p} > (1-g)v_H + gv_L$.

Low quality type uses the competitive advantage the lower prices give to engage in more aggressive manipulation than the high quality type. This in principle should increase the sales of the low quality type. However, low prices are also associated with higher demand.

Irrespective of which type would do more aggressive manipulation, the high quality type will always have the quality advantage in sales. This is stated by the following Proposition.

Proposition 2 *The profit level of H-type* π_H *is always higher than* π_L.

To isolate the impact of manipulative advertising on sales, we will compare the equilibrium sales when there is manipulative advertising and when the manipulative advertising is restricted to be 0 for both type at a fixed price. Specifically, we will tell that type j does effective manipulation at price $\bar{p}$ if

$$1 - F(\bar{x} - a_j - v_j) > 1 - F(\underline{x} - v_j) \Leftrightarrow a_j > \bar{x} - \underline{x}. \tag{12}$$

Recall that the threshold $\underline{x}$ is the signal that makes a consumer indifferent between purchasing and not purchasing the product, when there is no manipulation. It is uniquely defined by the equation (10). Therefore, $\bar{x} - \underline{x}$ is the average adjustment by consumers to manipulation by the Monopoly. If Monopoly of type j is doing more aggressive manipulation than the average adjustment by consumers, consumers fail to fully account for the manipulation by type j.

The map of effective manipulation over prices is closely related to the comparison between a_L and a_H that is laid out by Proposition 1. We show below that if type j does more aggressive manipulation than type k, then type j does effective manipulation. However, as intuitive as it may sound, effective manipulation by one type does not preclude the effective manipulation by the other type. Following Proposition 3 provides a map of effective manipulation.

Proposition 3 *Suppose that Assumptions 1, 2 and 4 hold. For cases in which* $\bar{p} < (1-g)v_H + gv_L$ *and* $\bar{x} > v_L + (a_L + a_H)/2$ *the only type that effectively manipulates is the L-type. For cases in which* $\bar{p} < (1-g)v_H + gv_L$ *and* $\bar{x} < v_L + (a_L + a_H)/2$ *both types can effectively manipulate.*

For the cases in which $\bar{p} > (1-g)v_H + gv_L$ and $\bar{x} \leq v_H + a_H$, the only type that effectively manipulates is the H-type. For the remaining cases, in which $\bar{p} > (1-g)v_H + gv_L$ and $\bar{x} > v_H + (a_L + a_H)/2$ both types can effectively manipulate.

Effective manipulation by type j is the net effect of manipulative advertising on sales of the type j Monopoly. If the only type that can effectively manipulate is the L-type, existence of manipulative advertising makes consumers worse-off, since the manipulative advertising increases the share of consumers who end up with a negative surplus if the Monopoly is of low quality type. If the only type that can effectively manipulate is the H-type, then consumers are better-off since the share of consumers who receive a positive surplus increases if the Monopoly is of H-type. To be more precise, we define the aggregate ex-ante surplus as follows:

$$(1-g)(v_H - \bar{p})(1 - F(\bar{x} - a_H - v_H)) - g(\bar{p} - v_L)(1 - F(\bar{x} - a_L - v_L)), \tag{13}$$

when there is manipulation. When there is no manipulation, ex-ante consumer surplus is defined as

$$(1-g)(v_H - \bar{p})(1 - F(\underline{x} - v_H)) - g(\bar{p} - v_L)(1 - F(\underline{x} - v_L)). \tag{14}$$

We define the net effect of the manipulative advertising on consumer surplus as the difference between these two types of consumer surplus at a fixed price $\bar{p}$.

Following Corollary 2, which is a direct result of Proposition 3, shows that the relatively lower prices are associated with negative effect of manipulative advertising and higher prices are associated with a positive effect of it.

Corollary 2 *The net effect of manipulative advertising on consumer surplus is negative when $\bar{p} < (1-g)v_H + gv_L$ and $\bar{x} > v_L + (a_L + a_H)/2$ and positive and $\bar{p} > (1-g)v_H + gv_L$ and $\bar{x} \leq v_H + a_H$.*

The cases that Corollary 2 leaves out are the ones where both types of Monopoly can do effective manipulation. In such cases, the net effect of manipulative advertising depends on the relative amount of manipulation by each type but also on the relative responsiveness of consumers to private signals. We provide a numerical example to show exactly how consumer surplus changes with manipulative advertising. We use normal distribution as the noise distribution, a quadratic function as the cost of manipulation and uniform distribution for the prior beliefs. Figure 3 illustrates that the net effect of manipulation is minimum at a price lower than the ex-ante expected value of the quality and maximum at a price higher than the ex-ante expected value of the quality.

Corollary 2 establishes that consumers may sometimes be better off if they simply received a noisy signal about product quality that is not biased through manipulative advertising. However, many times it may not be feasible (or it may be too costly) to acquire unbiased information about the quality of a product because informative messages are typically bundled together with biased statements, making it impossible to perfectly separate one from the other. In other words, if you choose to watch a TV commercial or browse the website of a company to get information about a newly released product, exposure to bias is typically a price you agree to pay. One natural question that pops up then is if you would rather ignore informative but biased advertising by switching the channel when a TV commercial appears or not visiting a web site that you know has an incentive to oversell a certain product. In a world where we are on a daily basis exposed to tons of implicit or explicit advertising content without our will, it is certainly hard, if not impossible, to insulate oneself from manipulative messages. Yet even if you somehow could ignore all these ads and choose not to receive any signal, it is still not straightforward if you would want to do it. Translated into our framework, the question is if a consumer would ever prefer to merely rely upon her prior beliefs when making a purchasing decision instead of acting upon her posterior beliefs after a manipulative advertising signal. The following proposition addresses this question.

Proposition 4 *Suppose that Assumptions 1, 2 and 4 hold. Then, a consumer would never ignore the only available advertising signal $x_i = v_j + a_j + \varepsilon_i$ (where $j \in H, L$ depending on the state) and base her purchasing decision on her prior beliefs.*

Proposition 4 establishes that no matter how high manipulative advertising by each potential type of monopolist is, consumers are better off by taking the advertising signals into account when making their decisions. At some level this result is to be expected, because despite the bias, advertising signals are informative about product quality. But at the same time, depending on the price level, manipulative advertising can worsen or improve ex-ante consumer surplus relative to non-manipulative

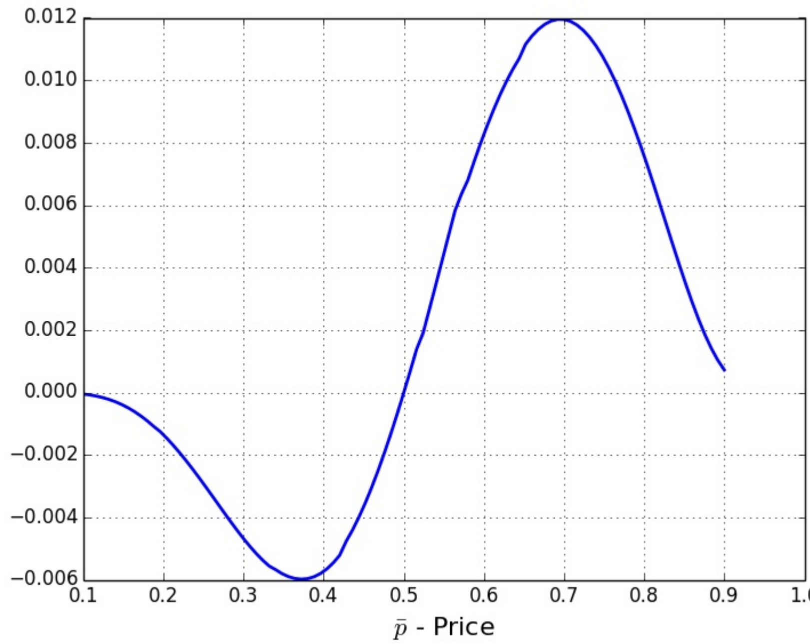

Figure 3: The Net Effect of Advertisement on Consumer Surplus

advertising. In this sense, Proposition 4 is not so straightforward. Without any signal, a consumer always purchases the product when $\bar{p} \leq gv_L + (1-g)v_H =: \tilde{p}$, and never purchases it otherwise. Interestingly, $\tilde{p}$ coincides with the price threshold below which L-type does greater manipulation than the H-type ($a_L > a_H$). Below this price, consumers overestimate quality when L-type is in charge and underestimate it when H-type is in charge. In contrast, when $\bar{p} > \tilde{p}$, manipulative advertising moves posterior beliefs in the welfare improving direction, but in the absence of any signal this is also when consumers (acting merely on their priors) will not purchase the product.

3.1 The Source of Consumers' Inference Problem

We show that depending on the price level and the state of the world (i.e. the type of the monopolist) manipulative advertising can improve or deteriorate consumers' posterior beliefs about product quality. Essential for this result is the presence of idiosyncratic noise in signals and the fact that two monopoly types employ different levels of advertisement in equilibrium.

When the monopolist with low product quality (L-type) advertises more than the monopolist with a high product quality (H-type), the median consumer will expect product quality to be higher [lower] when L-type [H-type] is in charge compared to what she would expect in the absence of any manipulation. This result follows from the fact that distance between signal means in each state will typically be different from the corresponding distance under the benchmark case where signals do not contain a systematic bias. To build some intuition, note that the random noise in signals has zero mean and has a unimodal and symmetric distribution. Monopolists can move the signal mean upward by an amount a through manipulative advertising at some cost $C(a)$. Imagine that the state is realized and consumers face an L-type. Consider the median consumer who would receive an advertising signal of $v_L + a_L$ when manipulation is allowed, and v_L when it is not allowed. Due to the noisy nature of the signals she cannot tell for sure if her signal comes from an L-type or an H-type monopolist. She instead uses her prior beliefs about the types and her knowledge of the noise distribution to assign posterior weights on these two events. If $a_L > a_H$, mean signal levels for the two types will be closer to each other compared to the benchmark situation where distance between mean signals is $\varepsilon = v_H - v_L > (v_H + a_H) - (v_L + a_L) = \varepsilon - (a_L - a_H)$. As a result, under manipulative advertising, the absolute magnitude of the random noise that would move a signal from $v_H + a_H$ to $v_L + a_L$ is smaller than the corresponding noise that moves a signal from v_H to v_L. Since the density of the noise distribution is symmetric around zero, the median consumer assigns a greater likelihood to the event that her signal comes an H-type when signals are manipulated such that $a_L > a_H$. When true type is H, by the same reasoning, she would assign greater likelihood to the event that her signal comes from an L-type. This inference problem would lead to a qualitatively opposite result if $a_L < a_H$.

4 Policy Analysis

There are various ways a regulator might want to intervene to the market to increase consumer surplus or any other measure of welfare. There are two broad categories of policy intervention based on the information requirements to the regulator.

If the regulator is able to afford collecting information about the quality of the product, the low quality monopoly type might specifically be targeted. One example of such policy is sampling the consumer complaints and devise a fine schedule for the monopoly based on some summary statistics of the complaints (see Drugov and Troya-Martinez (2015) for example of such a policy). In such a case, the low quality monopoly might face additional costs to manipulation. Such a policy would reduce the manipulation level that the low quality monopoly engages in without affecting the manipulation level by the high quality monopoly. Since the manipulation by the low quality monopoly is the source of any loss in the consumer surplus, such a policy is expected to increase consumer surplus.

A more crude intervention could be to enhance quality standards, where the regulator monitors the production process of the monopoly itself and prevents the monopoly to produce a low quality product. Such a policy would solve the information problem in the market. The only monopoly type in the market would be the high quality type. Then the game turns into a simple bargaining game between the monopoly and the consumers. If the price does not change from its level before regulation, the consumer surplus would increase. However, if the monopoly gets to choose the price after such a regulation, sequential rationality in that case would imply that the monopoly would expropriate all of the consumer surplus by setting the highest possible price, v_H. Therefore, such a policy does not necessarily increase consumer surplus as manipulative advertising might increase consumer surplus in some cases, as illustrated by Figure 3.

One disadvantage of the policies that require information acquisition by the regulatory agency is that information acquisition is potentially costly and therefore might cause additional burdens to consumers through taxation. Any policy that does not require costly information acquisition by the regulation agency will have an effect on both types of monopoly. The challenge that the regulatory agency might face would be that any policy that reduces manipulative advertising for the low quality type might also harm the high quality monopoly. The welfare implications of such an intervention is not straightforward. We discuss the policy of introducing a fixed toll for manipulative advertising below. We argue that even if such a policy affects both types of monopoly, it has a more profound effect on the low quality type, since its profit level is always lower than the high quality type, as shown in Remark 2.

4.1 Fixed Cost of Manipulative Advertising

Suppose that a regulatory agency imposes a lump-sum tax which the monopolist has to pay if it chooses to advertise. Such a tax could be imposed on advertising companies. Then the tax would be reflected in the price the advertising company charges to the monopoly. A fixed cost of advertising would not alter the monopoly's preferential ordering among positive manipulation levels but rather deter the monopoly from engaging in advertising at all.

Let $\bar{c}$ be the fixed cost of advertising. Then the monopoly would pay $\bar{c}$ in addition to the variable cost of manipulation $C(a_j)$. The profit of the monopoly of type j would be

$$\bar{p}(1 - F(\bar{x} - a_j - v_j)) - \bar{c} - C(a_j),$$

while if the monopoly does not engage in manipulative advertising, its profit would be

$$\bar{p}(1 - F(\bar{x} - v_j)).$$

Note that the first order conditions for an interior solution will not identify the optimal advertising decision of the monopoly in all of the cases. When the fixed cost is exceedingly high compared to the profit gains of advertising, the optimal decision of the monopoly will be the corner solution of zero manipulation.

The Figure 4 illustrates the manipulation level of the monopoly, when there is a fixed cost. On the left panel, there are price levels that both types engage in manipulation. Note that for every price that the low type engages in manipulative advertising, the high type does manipulation as well. This is due to the fact that the high quality type always makes a higher profit than the low type. Therefore, if the fixed cost does not deter the low type from doing advertising, then it should not deter the high type either. When we increase the fixed cost level a little bit further, the low type ceases to engage in manipulation. Therefore, there is a minimal level fixed cost $\bar{c}$ that completely deters the low type from manipulation, while allowing the high type to continue to do manipulation.

Figure 5 illustrates the impact of fixed cost on consumer surplus. Figure 5 depicts the change in the consumer surplus as we increase the fixed cost for low price (left-panel) and high price (right panel). If the fixed cost is high enough (higher than 0.4 in the Figure 5), the manipulation level is zero for both types. The effect of such a policy compared to the case, where there is no fixed cost, depends on the price level. If the price is low, as in the left-panel, the net effect of exceedingly high fixed cost is positive. However, if the price is high such a policy shuts down manipulation by the high-quality type more than it does to low-quality type. Irrespective of the price, the optimal fixed-cost level is always such that it is high enough to target the low-quality

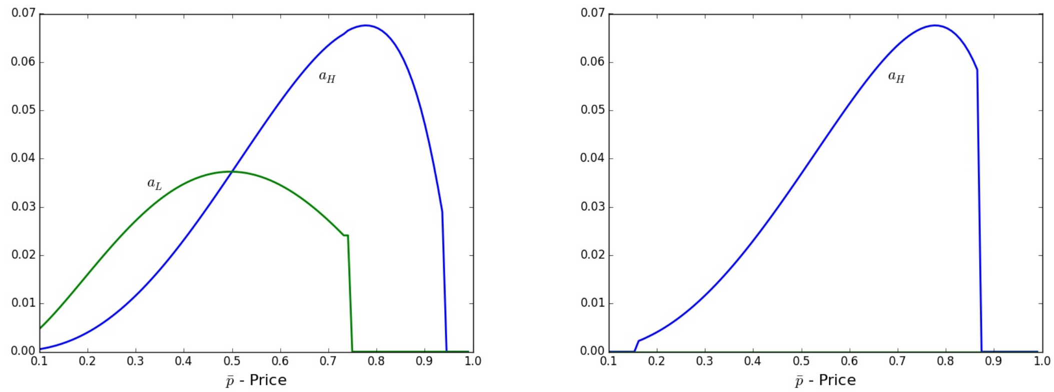

Figure 4: Manipulation Levels When There is Fixed Cost

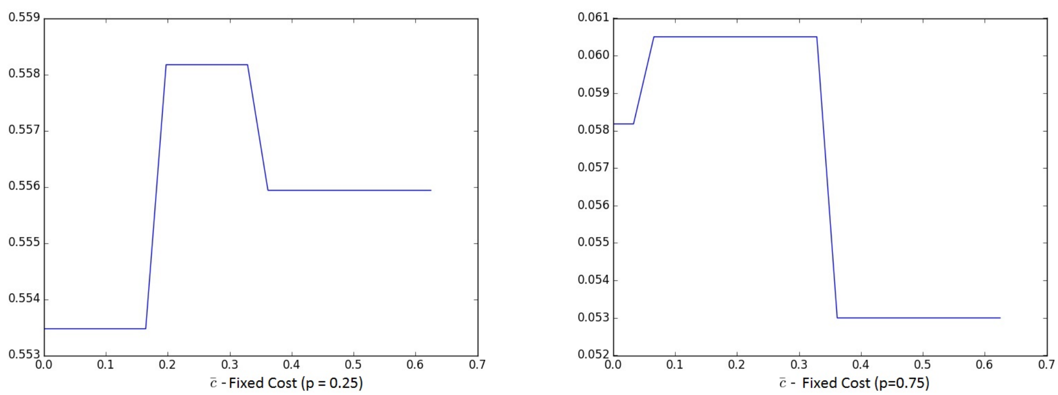

Figure 5: Effect of Increasing Fixed Cost on Consumer Surplus

level but not high enough to target the high quality as well.

5 Extensions

5.1 Ex-Ante Choice of Advertisement Levels

We have assumed so far that the monopoly chooses the advertisement level after it observes the quality of the product. This assumption is reasonable for the cases where firms invest in production taking before making decisions about marketing. However, analyzing the ex-ante choice of advertisement levels provides further insights about the strategic constraints the monopoly faces in the model that we describe above. Moreover, analysis in this section enables us to compare manipulative advertising to

Bayesian persuasion models in the literature.

Suppose that the monopoly has to choose a contingent advertisement plan given a common pooling price $\bar{p}$. That is, the monopoly chooses a couple of advertisement levels, $a_L, a_H \in [0,\infty)^2$ and commits to implement them after the quality of the product is realized. After, the product quality is realized, consumers observe the contingent advertisement plan by the monopoly and the private noisy signal about the quality level. Based on the information they have, consumers' form their posterior beliefs about the quality and decide whether to purchase the product.

There is no change in consumers' response to information manipulation. Therefore Corollary 1 extends to the ex-ante choice as well. For every information manipulation decision, there is a unique symmetric response by consumers, which can be characterized by the signaling threshold $\bar{x}$. For notational simplicity, we assume that the prior distribution is uniform and $v_L = 0$ for the discussion below.

The signaling threshold $\bar{x}$ satisfies equation (10), which we replicate here for convenience:

$$\frac{v_H f(\bar{x} - v_H - a_H)}{f(\bar{x} - v_H - a_H) + f(\bar{x} - a_L)} = \bar{p}. \tag{15}$$

Implicit function theorem provides us a continuously differentiable function $\bar{x}(a_L, a_H)$ for consumers' response. This actually guarantees the existence of a symmetric pure-strategy equilibrium. Moreover, $\bar{x} \to (-)\infty$ as $\bar{p} \to (0)v_H$.

The monopoly will maximize the expected profit given the prior beliefs. Monopoly's profit maximization is equivalent to maximizing the following:

$$\bar{p}(1 - F(\bar{x} - a_H - v_H)) - C(a_H) + \bar{p}(1 - F(\bar{x} - a_L)) - C(a_L). \tag{16}$$

Interior first order conditions are exactly as the ones in the ex-post advertisement choice model. However, Proposition 5 below implies that the local maximum found by these conditions is not global maximum for the ex-ante expected profit of the monopoly. Instead, the profit maximizing advertisement plan requires a corner solution, where at least one of the advertisement levels should be zero.

There are two important strategic differences of ex-ante choice of advertisement levels from the ex-port choice that contributes to the result stated by Proposition 5. Firstly, the monopoly becomes a Stackelberg-leader, since it has the strategic power to manipulate the responses that consumers give to the advertisement levels in contrast to the previous case, where the information processing decision by the consumers and manipulation decision by the monopoly were simultaneous. Secondly, the two contingencies of product quality become complementary tools for the monopoly, whereas in the previous case, each type of the monopoly has to compete with other type version of itself.

The monopoly knows by common knowledge of rationality, that the consumers adjust their expectations for manipulation. Thus, if the monopoly reduces the advertisement levels uniformly without changing the difference between the two levels, consumers will respond by adjusting their expectations less for the manipulation. A revealed preference argument based on Proposition 5 then shows that one of the advertisement levels are always 0 in any equilibrium.

Proposition 5 *Suppose that Assumptions 1 and 2 and 4 hold. If the monopoly can ex ante commit to a particular advertisement plan that is contingent on product quality, the optimal plan will never feature strictly positive advertisement for both quality types.*

The monopoly has three types choices for advertisement plan: (a_L, 0), (0, a_H) or $(0, 0)$. The choice of the monopoly depends on the sign of the net marginal revenue of advertisement. The net marginal revenue for a_H given that $a_L = 0$ is

$$\bar{p}f(\bar{x} - a_H - v_H)\left(\frac{\partial \bar{x}}{\partial a_H} - 1\right) + \bar{p}f(\bar{x})\frac{\partial \bar{x}}{\partial a_H} - C'(a_H), \tag{17}$$

where $\frac{\partial \bar{x}}{\partial a_H}$ can be implicitly defined using equation (15) as follows:

$$\frac{\partial \bar{x}}{\partial a_H} = \frac{(v_H - \bar{p})f'(\bar{x} - v_H - a_H)}{(v_H - \bar{p})f'(\bar{x} - v_H - a_H) - \bar{p}f'(\bar{x})}. \tag{18}$$

The net marginal revenue for a_L given that $a_H = 0$ is

$$\bar{p}f(\bar{x} - v_H)\frac{\partial \bar{x}}{\partial a_L} + \bar{p}f(\bar{x})\left(\frac{\partial \bar{x}}{\partial a_L} - 1\right) - C'(a_L), \tag{19}$$

where $\frac{\partial \bar{x}}{\partial a_L}$ can be implicitly defined using equation (15) as follows:

$$\frac{\partial \bar{x}}{\partial a_L} = \frac{\bar{p}f'(\bar{x} - a_L)}{\bar{p}f'(\bar{x} - a_L) - (v_H - \bar{p})f'(\bar{x} - v_H)}. \tag{20}$$

The monopoly calculates the optimal a_H and the corresponding $\bar{x}$ using the first order conditions (17), (18) and (15). Then calculates optimal a_L and the corresponding $\bar{x}$ using the first order conditions (19), (20) and (15). Then it calculates the profit levels at $(0,\ a_H,\ \bar{x}(0, a_H))$, $(a_L,\ 0,\ \bar{x}(a_L, 0))$ and finally $(0,\ 0,\ \underline{x})$. The advertisement plan of the monopoly is the triple among the three alternatives that maximizes its profit.

In contrast to the ex-post advertisement choice model, we cannot analytically rule out the no-advertisement as an equilibrium outcome. In some cases, the monopoly prefers simply not to intervene with the information distribution to the consumers. In particular, when $\bar{p} = v_H/2$, consumer indifference condition (15) requires $f(\bar{x} - a_L - v_L) = f(\bar{x} - a_H - v_H)$, which also implies that $f'(\bar{x} - a_L - v_L) = -f'(\bar{x} - a_H - v_H)$. This implies that marginal benefit of neither of the plans $(a_L, 0)$ nor $(0, a_H)$ lead to positive marginal benefit. Therefore, the monopoly chooses to do no manipulation when $\bar{p} = v_H/2$.

5.2 Information Precision

The dispersion of the private information of consumers depends negatively on the precision of the noise component of the signal they receive. When the noise component has a lower precision, it is more likely for consumers to receive an extreme signal. Therefore, the share of consumers who receive extreme signals is also higher. When the precision is high, consumers concentrate more around the mean signal, for which the noise term is zero. Precision has two effects on consumer behavior. The more precise their signals are, the more informative they are about the underlying quality of the product. Principally, it should be harder to deceive consumers when precision is high. However, when consumers are more likely to receive close messages, manipulative action of the monopoly may influence the posteriors of more consumers. To quantify these nuanced impact of precision, we adopt the following alteration of the private signal structure.

Given the quality level v_j and the manipulation level a_j each consumer i receives the private signal

$$x_i = v_j + a_j + \sigma \varepsilon_i,$$

where $\sigma > 0$ is the dispersion parameter. The higher σ is, the higher the dispersion among the consumers' private information about the quality is.

We can describe the equilibria indexed by price levels $\bar{p} \in (0, v_H)$ by using the same steps as before by making slight changes in the notation. The sales of the monopoly, given the threshold $\bar{x}$ that consumers use, is the probability that a consumer receives a signal that is greater than $\bar{x}$. That is,

$$Pr(x_i \geq \bar{x}|v_j) = Pr\left(\varepsilon_i \geq \frac{\bar{x} - v_j - a_j}{\sigma}\right) = 1 - F\left(\frac{\bar{x} - v_j - a_j}{\sigma}\right).$$

The interior solution to the maximization problem of the monopoly always exist with this framework as well. The first order condition for each type j is

$$\bar{p}f\left(\frac{\bar{x} - v_j - a_j}{\sigma}\right) = \sigma C'(a_j),$$

and the indifference condition for consumers that equates the expected quality and the price is

$$(v_H - \bar{p})f\left(\frac{\bar{x} - v_H - a_H}{\sigma}\right) = \bar{p}f\left(\frac{\bar{x} - a_L}{\sigma}\right).$$

The Figure 6 illustrates the effect of signal precision on the manipulation incentives of the monopoly. As precision of the private signal that the consumer receives increases, equivalently as the signal dispersion σ decreases, we first see a significant increase in the manipulation levels of both types of monopolies. As the signal precision increases consumers concentrate more around the median signal. This way same level of manipulation becomes more effective in moving the posteriors of consumers. This increases the marginal benefit of manipulation for the monopoly, therefore increases the equilibrium manipulation levels. However, as signal precision increases further the accuracy effect of the precision parameter dominates the concentration effect. Consumers receive extremely informative signals about the quality for high precision values. Therefore for every price either almost all consumers decide to buy the good or almost none of them buys the good. At these extreme demand conditions, the marginal impact of manipulation decreases, and so do the manipulation levels.

6 Endogenous Price

We have assumed so far that the price, being an exogenous parameter, did not carry any information about the quality of the product. When the monopoly gets to choose the price and the advertisement levels, the monopoly may in principle choose price in an informative (or a misleading) way. The informative power of price in addition to

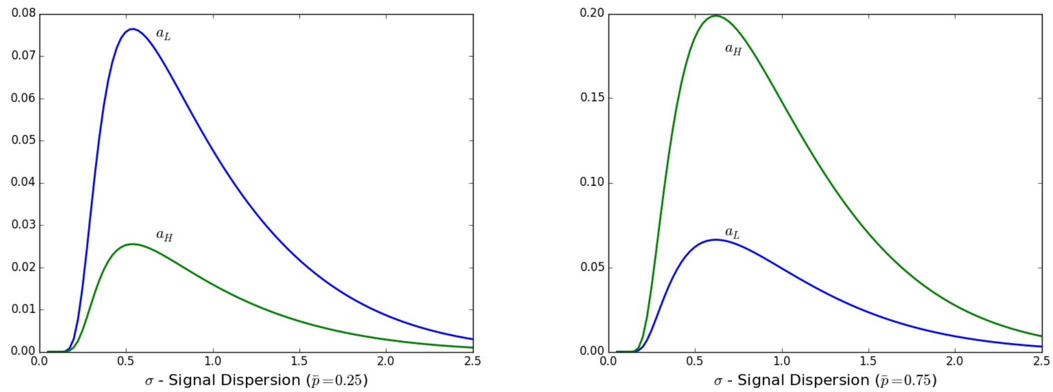

Figure 6: Manipulation Levels as Signal Dispersion Increases

the noisy signals that consumers receive depends on the equilibrium coordination of expectations.

In this section, we will first show that the sets of equilibria are qualitatively different when $v_L = 0$ and $v_L > 0$. When $v_L = 0$, every price between 0 and v_H can be supported as a pooling equilibrium. In addition to these strict equilibria, where manipulation levels and the profits are positive, there are also weak pooling and separating equilibria, where it is possible to observe a price that is at least as high as v_H and no type of monopoly makes any sales.

When $v_L > 0$, on the other hand, the weak pooling and separating equilibria cease to exist. Moreover, the prices that are close to v_H cannot be supported by pure-strategy equilibria. However, we show that if we allow the monopoly to mix between prices, we find that these higher prices can be supported by partially-separating mixed equilibria. In these equilibria, L-type mixes between v_L and some other price $\bar{p} \in (v_L, v_H)$, and H-type chooses the price $\bar{p}$ as a pure strategy. Since, $\bar{p}$ carries some imperfect information, consumers employ both the price and the noisy signals as sources of information. We discuss the properties of these equilibria below.

We employ symmetric Perfect Bayesian Equilibrium (PBE) as the solution concept for this section as well. Intuitively, PBE requires sequential rationality and Bayesian update for posterior beliefs whenever possible. We restrict our attention to the pure advertising strategies in this section as well; however, we allow for mixed pricing strategies, of which support lies in the interval $[v_L, v_H]$.

To simplify the notation throughout the analysis, we will assume that the consumers' prior beliefs assign equal probability to both monopoly types. A strategy profile is the advertisement choice of each type of monopoly (a_L, a_H), (possibly mixed) pricing decision (β_L, β_H), and the purchasing decision of the consumer $s(x,p)$ after observing the noisy signal x and price p . Then a strategy profile

$$\langle a_L^*, a_H^*, \beta_L, \beta_H, s^*(\cdot,\cdot)\rangle$$

accompanied with the posterior belief of a consumer $\mu(x,p)$ who observed the signal x and price p is a symmetric pure-strategy PBE if and only if the strategy by the monopolist is defined as

$$\begin{aligned}
a_L^* &\in \operatorname*{argmax}_{a_L\in[0,\infty)} p_L S(a_L, a_H^*, p_L, p_H, v_L) - C(a_L) \\
a_H^* &\in \operatorname*{argmax}_{a_H\in[0,\infty)} p_H S(a_L^*, a_H, p_L, p_H, v_H) - C(a_H) \\
\beta_L &\in \operatorname*{argmax}_{\beta_L\in\Delta([v_L,v_H])} \int_{p\in supp(\beta_L)} pS(a_L, a_H^*, \beta_L, \beta_H, p, v_L) - C(a_L) \\
\beta_H &\in \operatorname*{argmax}_{\beta_H\in\Delta([v_L,v_H])} \int_{p\in supp(\beta_H)} pS(a_L, a_H^*, \beta_L, \beta_H, p, v_L) - C(a_H),
\end{aligned} \tag{21}$$

and the strategy and beliefs of consumers are defined as

$$\begin{aligned}
S(a_L, a_H^*, \beta_L, \beta_H, p, v_L) &= \int_{p\in supp(\beta_L)} \int_{-\infty}^{\infty} s(x,p) f(x - v_j - a_j^*) dx \\
S(a_L, a_H^*, \beta_L, \beta_H, p, v_H) &= \int_{p\in supp(\beta_H)} \int_{-\infty}^{\infty} s(x,p) f(x - v_j - a_j^*) dx \\
s(x,p) &= \begin{cases} 1 & \text{if} \quad \sum_j (v_j - \bar{p})\mu(x,p)(v_j) \geq 0 \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{22}$$

and $\mu(x,p)$ is formed by Bayesian update whenever possible.

6.1 Analysis

In contrast to the analysis in Section 2, we cannot write down the posterior beliefs $\mu(x,p)$ as we did with posterior expectations with exogenous prices in equation (9). We first need to specify how informative is the pricing decision of the monopoly. The Proposition 6 below states that there are no pure strategy separating equilibria that supports positive profit level.

Proposition 6 *If $v_L = 0$, there are pure strategy separating equilibria, but in all of them the sales of the high quality type is* 0*.*

If $v_L > 0$, there are no pure strategy separating equilibria.[6]

The following Proposition 7 states that there is a set of pure-strategy pooling equilibria that coincides with the exogenous price cases that we analyzed in section 2.1

Proposition 7 *Suppose that Assumptions 1, 2 and 4 hold. If $v_L = 0$, for every price $\bar{p} \in (0, v_H)$ there is a pooling equilibrium that supports $\bar{p}$ as the equilibrium price. The equilibrium strategies, except pricing, are determined via the equilibrium conditions laid out in Theorem 1.*

If $v_L > 0$, prices that are close enough to v_H cannot be supported by a pooling equilibrium.

Following Theorem 2 states that higher prices could be supported by a mixed strategy equilibrium.

Theorem 2 *Suppose that Assumptions 1, 2 and 4 hold. There exists a pricing threshold $\tilde{p} \in (v_L, v_H)$ such that for every price $\bar{p} \in (\tilde{p}, v_H)$, there exists a partially separating equilibrium. On this equilibrium, the low quality type chooses the manipulation level $a_L > 0$ and plays a binary mixed strategy that puts probability $\bar{\alpha}(\bar{p})$ to price $\bar{p}$ and $1 - \bar{\alpha}$ to the price v_L. High quality type always chooses price $\bar{p}$ and chooses the manipulation level $a_H > 0$. Each consumer i, who receives the signal x_i purchases the product either when the observed price is v_L or when the signal $x_i \geq \bar{x}$, for some signaling threshold $\bar{x}$.*

On this equilibrium, the mixing probability $\bar{\alpha}$, the manipulative advertising levels a_L, a_H, and the signaling threshold $\bar{x}$ satisfy the following equation system:

$$v_L = \bar{p}(1 - F(\bar{x} - a_L - v_L)) - C(a_L) \tag{23}$$

$$\bar{p} f(\bar{x} - a_L - v_L) = C'(a_L) \tag{24}$$

$$\bar{p} f(\bar{x} - a_H - v_H) = C'(a_H) \tag{25}$$

$$(v_H - \bar{p}) f(\bar{x} - a_H - v_H) = \bar{\alpha}(\bar{p} - v_L) f(\bar{x} - a_L - v_L). \tag{26}$$

[6]This result would not change if H-type had a higher fixed marginal cost of production as well.

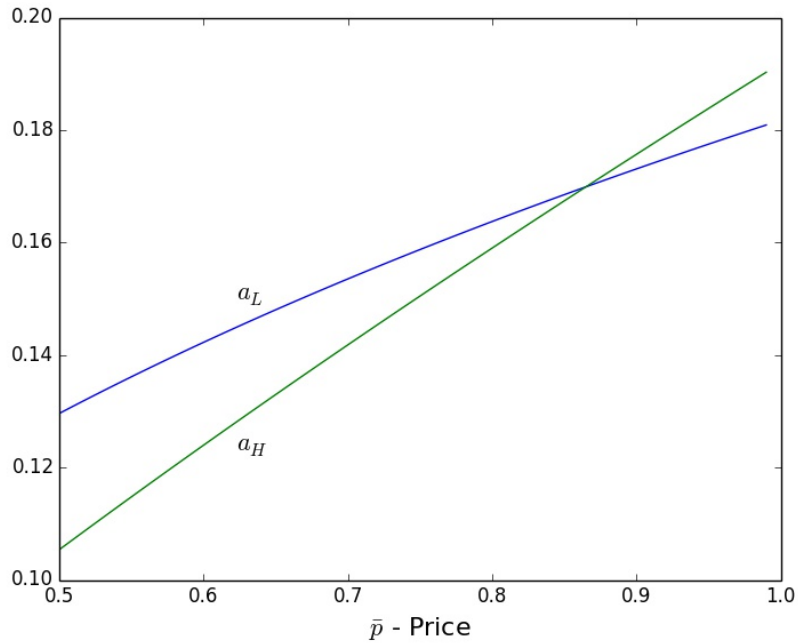

Figure 7: Manipulation Levels as Price Increases

Comparative statics with respect to price on the path of partially separating equilibria have distinctive features compared to the path of pooling equilibria. Following Proposition 8 states some of the interesting effects of price on the equilibrium behavior.

Proposition 8 *Consider the set of partially separating equilibria, which are described in Theorem 2 and indexed by the interval of prices* $(\tilde{p}, v_H)$*. Over this equilibrium path, the signaling threshold* $\bar{x}$*, manipulation level* a_L *by* L*-type and the profit of* H*-type increase with price. That is,* $\partial\bar{x}/\partial\bar{p}$*,* $\partial a_L/\partial\bar{p}$*, and* $\partial\pi_H/\partial\bar{p} > 0$*.*

Proposition 8 states that the manipulative advertising level by L-type increases with the price level. Figure 7 shows that both manipulation levels increase with price. Moreover, for extremely high prices H-type starts to engage in more aggressive manipulation than L-type. This is also stated in the Proposition 9 below.

Proposition 8 shows two opposing effects of price on the path of partially separating equilibria. On the one hand, $\partial\bar{x}/\partial\bar{p} > 0$, which means that the consumers are harder to be convinced by their private signals when the prices are higher. Since the price increases as well, one would expect that the demand is lower. However, Proposition 8 also states that $\partial\pi_H/\partial\bar{p} > 0$, which means that the profits of the H-type increases

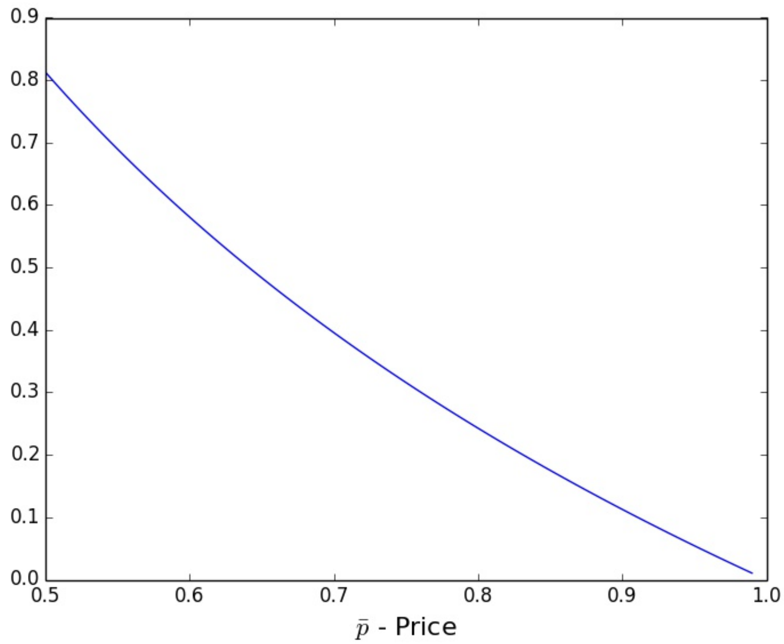

Figure 8: $\bar{\alpha}$ - Mixing Probability as Price Increases

as price increases. The reason that profits increase cannot be attributed to decrease in advertising costs, since advertising levels increase with price as demonstrated in Figure 7.

The increase in the profit with price is due to the quality of the public signal that consumers receive. A higher public signal; that is, a higher price $\bar{p}$ translates to the inference that consumers assign a higher likelihood to a H-type monopolist. To see this, we need to look at the behavior of the mixing probability $\bar{\alpha}$ as the price increases. Figure 8 shows that the mixing probability $\bar{\alpha}$ decreases as price $\bar{p}$ increases. As the price increases, the revenues for higher price increases for the L-type. To counterbalance this effect, L-type reduces the mixing probability $\bar{\alpha}$ for higher price. This increases the pre-signal likelihood for consumers that the monopoly is of H-type, when consumers observe the high price $\bar{p}$.

6.1.1 Effective Manipulation

When the monopoly is not allowed to do manipulative advertising, consumers consult to their private signals and the pricing to infer the quality of the product. When they observe v_L as the price, they are sure that the monopoly is of L-type.

When consumers observe $\bar{p}$, they remain uncertain about the quality. The equilibrium conditions for a partially separating equilibrium when manipulation is not allowed are as follows:

$$v_L = \bar{p}(1 - F(\underline{x} - v_L)) \tag{27}$$

$$(v_H - \bar{p})f(\bar{x} - v_H) = \underline{\alpha}(\bar{p} - v_L)f(\bar{x} - v_L), \tag{28}$$

where $\underline{\alpha}$ is the mixing probability that monopoly of type L plays $\bar{p}$, when there is no manipulation and $\bar{p}$ is sufficiently large that there is a partially-separating equilibrium both when there is manipulation and not.

It is straightforward to show for prices that are sufficiently close to v_H that a partially separating equilibrium without manipulation described by equations (27) and (28) exist. To show the impact of the manipulative advertising we compare the sales of the monopoly for a given price $\bar{p}$ when there is advertising and not. In particular, we check for each type j

$$1 - F(\bar{x} - a_j - v_j) > 1 - F(\underline{x} - v_j) \Leftrightarrow a_j > \bar{x} - \underline{x}.$$

Following Proposition characterizes the cases in which the H-type does a more aggressive manipulative advertising than the L-type and states that the manipulative advertising is effective for both types.

Proposition 9 *The manipulation levels by both types are equal to each other; that is, $a_L = a_H$ if and only if*

$$\bar{p} = \frac{v_H + \bar{\alpha} v_L}{1 + \bar{\alpha}} =: \hat{p}_{\bar{\alpha}}. \tag{29}$$

$a_H > a_L$ *if and only if* $\bar{p} > \hat{p}_{\bar{\alpha}}$.

The manipulative advertising is always effective for the L-type and is effective for H-type when $\bar{p} \geq \hat{p}_{\bar{\alpha}}$.

Recall that for the pooling equilibria when $a_L = a_H$, no type of monopoly could effectively manipulate consumers. Consumers' response to manipulation was exactly discounting the effect of manipulation; that is, $\bar{x} - \underline{x} = a_L = a_H$. However, for the partially separating equilibria, when $a_L = a_H$, both types can effectively manipulate. The main difference between the two types of equilibria that causes this difference is

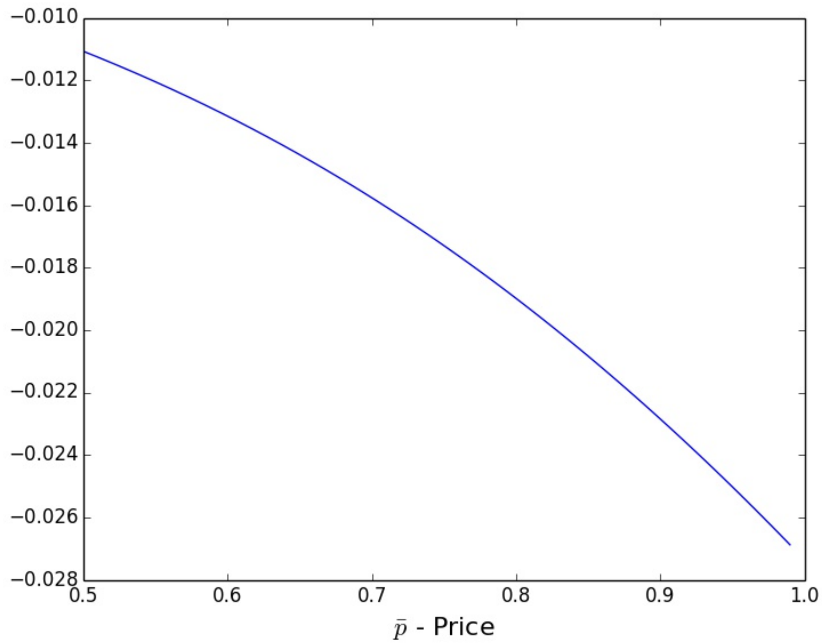

Figure 9: Net Effect of Manipulation on Consumer Surplus

the informative function of the price. For the partially separating equilibria, even if we keep the price same, when we close the channel of manipulative advertising, the equilibrium mixed strategy of the L-type changes from $\bar{\alpha}$ to $\underline{\alpha}$. Therefore, the sales of both types of the monopoly increases when manipulative advertising is introduced exactly because of the change in the mixing probability α. As the mixing probability changes, the consumers become more convinced that the product is of high quality. Therefore, even if they discount the impact of manipulative advertising, their response is still to increase their likelihood of purchase.

One of the different implications of informative pricing from uninformative one is that the low quality monopolist can always increase its market sales through manipulative advertising over the path of partially separating equilibria, as stated in Proposition 9. This is in contrast to the case of uninformative pricing, where effective manipulation by L-type is possible only in relatively low prices, as stated in Proposition 3. Therefore, the net effect of manipulative advertising on consumer surplus is also different, when the prices are informative. Figure 9 shows that manipulative advertising always reduces consumer surplus compared to the case where there is no manipulative advertising.

The policy implication of this negative welfare effects of manipulative advertising, is then more straightforward. A regulator may always introduce a non-zero fixed cost for manipulative advertising to increase the consumer surplus.

6.2 Equilibrium Selection

Proposition 7 and Theorem 2 show the existence of two sets of equilibria indexed by the price levels. One of the implications of these two results is that the pricing outcome of the game is resolved through the coordination between the expectations of the monopoly and consumers. In particular, the off-equilibrium path beliefs of the consumers for each equilibrium prevent the monopoly to deviate to a price other than the one prescribed by the equilibrium. The most conservative off-equilibrium beliefs that support the equilibria are such that whenever consumers observe a price other than what is prescribed by the equilibrium, they believe that it is only the low-quality type that could make such a deviation. Therefore, they would not purchase the good for any price $\bar{p} > v_L$. These extreme beliefs support any type equilibrium described by Proposition 7 and Theorem 2. However, some of the equilibria can be supported by less extreme off-equilibrium beliefs as well. In this section, we investigate which subset of equilibria survives when we put some discipline in the off-equilibrium beliefs.

Firstly, observe that the assumption that production cost is same for both types makes it impossible for the high quality type to credibly communicate to the consumers that it is not a low-quality type. This is because if the high quality expects to get a higher profit by deviating to an off-equilibrium price, it expects that consumers would have favorable enough expectations. However, the low-quality can simply imitate the same behavior as well, therefore from the consumers' perspective if an off-equilibrium price offers a higher profit to one type, it should offer the same for all types. This immediately implies that the Intuitive Criterion cannot eliminate any equilibria, even if it is a weak one. Indeed a straightforward check of definition of Intuitive Criterion proves the following result.

Proposition 10 *All PBE satisfy the requirements of Intuitive Criterion.*

The reason that Intuitive Criterion does not help us refining the set of equilibria is that it is impossible to devise credible deviations that benefits only one of the types. Therefore, the only way we can eliminate some equilibria is by arguing that both types can benefit from deviating to an off-equilibrium price. For such an argument we employ the concept of "Undefeated Equilibrium". We first make some observations about off-equilibrium beliefs that consumers can have that support the equilibria.

To simplify the analysis for equilibrium selection among the pooling equilibria in what follows we assume that the ex-post profit functions are strictly concave in price level and, therefore, there is an "optimal" price for each type of monopoly that maximizes the ex-post profit for each type among the pooling equilibrium profit levels. This property enables us to rank the pooling equilibria for each type.

Assumption 5 *Let $\Phi \subseteq (v_L, v_H)$ be set of prices that could be supported by Pooling PBE. The equilibrium profit functions are strictly concave in prices and for each type $j \in \{L, H\}$ there exists a unique price $\bar{p}_j \in \Phi$ such that for any other price $p \in \Phi$*

$$\pi_j(a_L^*(\bar{p_j}), a_H^*(\bar{p_j}), \bar{x}(\bar{p}_j)) \geq \pi_j(a_L^*(p), a_H^*(p), \bar{x}(p)).$$

Following Lemma proves that the best price p_H for the high type is higher than the best price p_L for the low type.

Lemma 3 *Suppose that Assumption 5 holds. The price p_H that gives the highest profit among the pooling equilibria to H-type is strictly greater than the corresponding price p_L for the L-type.*

One implication of Lemma 3 is that there is an interval $[p_L, p_H]$ such that for any $p \in [p_L, p_H]$ and $p' \notin [p_L, p_H]$,

$$\pi_j(a_L^*(p), a_H^*(p), \bar{x}(p)) > \pi_j(a_L^*(p'), a_H^*(p'), \bar{x}(p'))$$

for each type $j \in \{L, H\}$.

Second implication of Lemma 3 is that there is no unique price that makes a pooling equilibrium the best one for both types. Therefore, for any pooling equilibrium, each type should face a less favorable out-of equilibrium belief by consumers. Since otherwise, at least one of the types could deviate to the price at which the best equilibrium supports. Based on this argument, following Proposition 11 provides two "bounds" for off-equilibrium beliefs that support pooling equilibria.

Proposition 11 *Let $\bar{p} \in (v_L, v_H)$ be any pooling equilibrium price. Then the following belief about the informativeness of prices is sufficient to support a pooling equilibrium.*

$$P(v = v_H | p) = \begin{cases} P(v = v_H) & if \quad p = \bar{p} \\ 0 & o.w. \end{cases} \tag{30}$$

On the other hand, the following belief is necessary for any pooling equilibrium that supports $\bar{p}$. There exists a range of prices $R(\bar{p}) \subset (v_L, v_H)$ such that for any price $p \in R(\bar{p})$.

$$P(v = v_H | p) \begin{cases} = P(v = v_H) & if \quad p = \bar{p} \\ < P(v = v_H) & o.w. \end{cases} \tag{31}$$

One of the implications of Proposition 11 is that whenever consumers observe an off-equilibrium price, they should reduce the likelihood of a high quality monopolist from the prior belief. If this is not the case for a particular price, that price cannot be supported by a pooling equilibrium.

Next, we analyze below how much "Undefeated Equilibrium" might help in refining the set of equilibria. The original definition proposed by Mailath, Okuno-Fujiwara and Postlewaite (1993) was designed for single signal sender and a receiver. We extend the definition for the information manipulation model with many receivers, additional private information to receivers and mixed-pricing strategies as below.

Definition 1 *An equilibrium ($\hat{a}_L$, $\hat{a}_H$, $\hat{\beta}_L$, $\hat{\beta}_H$, $\hat{s}(x,p)$, $\hat{\mu}(x,p)$) defeats ($\tilde{a}_L$, $\tilde{a}_H$, $\tilde{\beta}_L$, $\tilde{\beta}_H$, $\tilde{s}(x,p)$, $\tilde{\mu}(x,p)$) if there exists a price $p' \in [v_L, v_H]$ such that*

1. *for any type $j \in \{L, H\}$ of monopoly, $p' \notin supp(\beta_j)$ and $K = \{j \in \{L, H\} | p' \in supp(\beta_j)\} \neq \emptyset$;*

2. *for any type $j \in K$ the profit of type j-monopoly $\pi(\tilde{\beta}_j,\ \tilde{a}_j,\ \tilde{s}(\cdot,\cdot)) \geq \pi(\hat{\beta}_j,\ \hat{a}_j,\ \hat{s}(\cdot,\cdot))$ and for at least one of the types $j' \in K$ $\pi(\tilde{\beta}_{j'},\ \tilde{a}_{j'},\ \tilde{s}(\cdot,\cdot)) > \pi(\hat{\beta}_{j'},\ \hat{a}_{j'},\ \hat{s}(\cdot,\cdot))$;*

3. *there exists a type $j \in K$*

$$\tilde{\mu}(x, p')(j) \neq \frac{P(x|j)P(j)\phi(j)}{P(x|L)P(L)\phi(L) + P(x|H)P(H)\phi(H)},$$

for any function $\phi : \{L, H\} \to [0, 1]$ satisfying

$$
\begin{aligned}
j' \in K \, and \quad \pi(\tilde{\beta}_{j'}, \tilde{a}_{j'}, \tilde{s}(\cdot,\cdot)) > \pi(\hat{\beta}_{j'}, \hat{a}_{j'}, \hat{s}(\cdot,\cdot)) \quad &\Rightarrow \phi(j') = 1, \\
j' \notin K \qquad\qquad\qquad\qquad\qquad\qquad &\Rightarrow \phi(j') = 0.
\end{aligned}
$$

An equilibrium is called undefeated if and only if it is not defeated by another equilibrium.

The relation of defeating constitutes a strict partial order among the PBE. It is possible to rule out some of the outcomes using this order. However, due to the multiplicity of off-equilibrium beliefs that may support the same outcome, it is not possible to select a single equilibrium outcome or even one type of outcome. We show that a strict subset of pure strategy pooling equilibria (described in Proposition 7) are undefeated and that partially separating equilibria (described in Theorem 2) that have relatively conservative off-equilibrium beliefs can all be defeated by another partial-separating equilibria. Moreover, pooling equilibria and partially-separating equilibria cannot be compared by the defeating relationship.

Following Proposition 12 shows that the interval of prices $[\bar{p}_L, \bar{p}_H]$, which are specified by Assumption 5 are undefeated.

Proposition 12 *Suppose that Assumptions 1, 2, 3, and 5 hold. A pooling equilibrium is undefeated only if it supports a price $\bar{p} \in [\bar{p}_L, \bar{p}_H]$. Moreover pooling equilibria that support a price in the interval $[\bar{p}_L, \bar{p}_H]$ is not defeated by another pooling equilibria.*

The following Proposition 13 shows that we can rank partially separating equilibria based on the price it supports if they have rather conservative off-equilibrium beliefs.

Proposition 13 *Consider any partially separating equilibrium that supports a price $\bar{p} \in (v_L, v_H)$. If the off-equilibrium probability $P(v_H|p') < 0.5$ for any price $p' \neq \bar{p}$, there is another partial separating equilibrium that supports $\hat{p} > \bar{p}$ and defeats the equilibrium that supports $\bar{p}$.*

7 Conclusion

We study the implications of manipulative advertising by a monopolist under asymmetric information and a noisy communication environment. We have modeled advertising as a signal jamming technology and shown that the ability to increase demand via advertising is not reserved for only one type of monopolist. Depending on the price level, both types can be effective manipulators. Under relatively low prices, the low-quality monopolist has more incentive to advertise than the high-quality monopolist and the signal jamming technology delivers higher sales for its product. The opposite is true when prices are relatively high. Beyond its informational consequences, which, depending on the context, might be harmful or beneficial for consumers, manipulative ads lead to an arms race between the two types of monopolists and socially wasteful spending.

Our work provides a novel rationale for regulating advertisement spending. In particular, we show that there is an optimal fixed cost of advertising, where a regulating agency may impose on advertising agencies or firms, that filters harmful manipulative advertising from a beneficial one. Fixed cost of advertising is one of the simplest advertisement regulations. There are, however, more sophisticated policy tools regarding quality control, customer protection and so on, which could complicate the information structure by making many public and private signals accessible to the consumers. Our model, which relies on the framework that consumers receive a single private information, may not capture these cases.

We find that the two types of a monopolist enter in an implicit competition among each other. This results in wasteful advertisement spending by the monopolist to the point that the net effect of advertisement on expected profit might be negative. This result depends in part on the way the price is determined in the equilibrium. In our model, price is determined through the coordination of expectations between the monopolist and the consumers. Nevertheless, in markets with multiple firms, the downward pressure on prices might break down this coordination of expectations. Analysis of duopolies and oligopolies when firms have access to manipulative advertisement remains a challenging and interesting open question.

A Proofs

Proof of Lemma 1 Suppose first that Assumption 3 holds. Then, each first order condition (7) has a unique solution, since Assumption 3 guarantees the strict concavity of the profit function globally.

Suppose for a moment that $a_L + v_L \geq a_H + v_H$, which would imply that $\bar{x} - a_L - v_L \leq \bar{x} - a_H - v_H$ and also $a_L > a_H$. Then having a unique solution to equation (7) implies that if L-type uses the lower manipulation level a_H, its marginal revenue should be greater than its marginal cost. That is,

$$\bar{p}f(\bar{x} - a_H - v_L) > C(a_H) = \bar{p}f(\bar{x} - a_H - v_H),$$

which implies

$$|\bar{x} - a_H - v_L| < |\bar{x} - a_H - v_H|,$$

since the noise pdf $f(\cdot)$ is unimodal. Then, $v_L < v_H$ implies that $\bar{x} > a_H + v_H$. On the other hand, $a_L > a_H$ implies by first order conditions (7) that

$$f(\bar{x} - a_L - v_L) > f(\bar{x} - a_H - v_H) \Leftrightarrow |\bar{x} - a_L - v_L| < |\bar{x} - a_H - v_H|,$$

which implies $\bar{x} < a_H + v_H$. Hence, a contradiction.

Now, suppose that Assumption 4 holds and $a_L + v_L \geq a_H + v_H \Leftrightarrow a_L > a_H + v_H - v_L$. But at such a manipulation level by the L-type, the marginal cost would always be greater than the marginal revenue by Assumption 4.

Proof of Lemma 2 For notational simplicity, denote $f_j \equiv f(x - v_j - a_j)$ and $f'_j \equiv f'(x - v_j - a_j))$. Then,

$$\frac{\partial E[v|x]}{\partial x} = \frac{\sum_{j\in J} v_j f'_j G(v_j)\left(\sum_{j\in J} f_j G(v_j)\right) - \sum_{j\in J} f'_j G(v_j)\left(\sum_{j\in J} v_j f_j G(v_j)\right)}{\left(\sum_{j\in J} f_j G(v_j)\right)^2} > 0 \Leftrightarrow$$

$$\sum_{j\in J} v_j f'_j G(v_j)\left(\sum_{j\in J} f_j G(v_j)\right) - \sum_{j\in J} f'_j G(v_j)\left(\sum_{j\in J} v_j f_j G(v_j)\right) > 0 \Leftrightarrow$$

$$g(1-g)f'_H f_L \quad > \quad g(1-g)f_H f'_L \Leftrightarrow \frac{f'(\bar{x} - a_H - v_H)}{f(\bar{x} - a_H - v_H)} \geq \frac{f'(\bar{x} - a_L)}{f(\bar{x} - a_L)}, \qquad (32)$$

which holds, since f is log-concave by Assumption 1 and $a_H + v_H > a_L + v_L$ by Lemma 1.

Proof of Theorem 1 By Assumption 4 and Inverse Function Theorem, for each manipulative advertisement couple a_L, a_H there exists a unique $\bar{x}$, and $\bar{x}(a_L, a_H)$ is a continuously differentiable function. Therefore we can reduce the number of equations that define the equilibrium into the following two equations that are very similar to the first order conditions (7):

$$\bar{p} f(\bar{x}(a_L, a_H) - v_j - a_j) = C'(a_j) \quad \text{for all } j \in J, \tag{33}$$

which has a positive solution by Assumptions 1, 2, and Intermediate Value Theorem. Moreover, the solution is unique if Assumption 3 holds as well.

Lemma 4 *Suppose that Assumptions 1, 2 and 3 hold. When $\bar{p}$ converges to v_L, the signaling thresholds $\bar{x}$ and $\underline{x}$ diverge to $-\infty$, and when $\bar{p}$ converges to v_H, the signaling thresholds $\bar{x}$ and $\underline{x}$ diverge to ∞.*

Proof of Lemma 4 When $\bar{p} < (v_H + v_L)/2$, $a_L > a_H$ and by first-order conditions (7)

$$f(\bar{x} - a_L - v_L) > f(\bar{x} - a_H - v_H) \Leftrightarrow |\bar{x} - a_L - v_L| < |\bar{x} - a_H - v_H|,$$

since $f(\cdot)$ is unimodal. Then by Lemma 1 $\bar{x} < a_H + v_H$.

On the other hand, hen $\bar{p} > (v_H + v_L)/2$, $a_L < a_H$ and by first-order conditions (7)

$$f(\bar{x} - a_L - v_L) < f(\bar{x} - a_H - v_H) \Leftrightarrow |\bar{x} - a_L - v_L| > |\bar{x} - a_H - v_H|,$$

then by Lemma 1 $\bar{x} > a_L + v_L$.

The consumer indifference condition (11) can rewritten as follows

$$\frac{v_H - \bar{p}}{\bar{p} - v_L} = \frac{f(\bar{x} - a_L - v_L)}{f(\bar{x} - a_H - v_H)}.$$

When $\bar{p} \to v_L$, LHS of the equation above converges to ∞ and therefore $f(\bar{x} - a_H - v_H) \to 0$, which implies $\bar{x} \to \{-\infty, \infty\}$. But since $\bar{x} \leq a_H + v_H < \infty$, $\bar{x} \to -\infty$.

When $\bar{p} \to v_H$, LHS of the equation above converges to 0 and therefore $f(\bar{x} - a_L - v_L) \to 0$, which implies $\bar{x} \to \{-\infty, \infty\}$. But since $\bar{x} \geq a_L + v_L > -\infty$, $\bar{x} \to \infty$.

Proof of Proposition 2 The proof follows from a simple revealed-preference argument.

The profit level of L-type is

$$\bar{p}(1 - F(\bar{x} - a_L)) - C(a_L).$$

If the H-type imitated L-type, its profit would be

$$\bar{p}(1 - F(\bar{x} - a_L - v_H)) - C(a_L) > \bar{p}(1 - F(\bar{x} - a_L)) - C(a_L),$$

which implies the optimal profit level that the H-type can achieve is strictly higher than that of L-type.

Lemma 5 *Suppose that Assumptions 1, 2 and 3 hold. The signaling threshold $\bar{x}$ strictly increases with price $\bar{p}$. That is, $\partial\bar{x}/\partial\bar{p} > 0$.*

Proof of Lemma 5 To simplify the notation let $f(\bar{x} - a_L - v_L) = f_L$, $f(\bar{x} - a_H - v_H) = f_H$, $f'(\bar{x} - a_L - v_L) = f'_L$, $f'(\bar{x} - a_H - v_H) = f'_H$.

Implicit differentiation of first-order conditions (7) enables us to calculate $\partial a_L/\partial\bar{p}$ and $\partial a_H/\partial\bar{p}$ as follows.

$$\begin{aligned} f_L + \bar{p}f'_L\frac{\partial\bar{x}}{\partial\bar{p}} &= (\bar{p}f'_L + C''(a_L))\frac{\partial a_L}{\partial\bar{p}} \Leftrightarrow \\ \frac{\partial a_L}{\partial\bar{p}} &= \frac{f_L}{\bar{p}f'_L + C''(a_L)} + \frac{\bar{p}f'_L}{\bar{p}f'_L + C''(a_L)}\frac{\partial\bar{x}}{\partial\bar{p}} \quad \text{and,} \\ f_H + \bar{p}f'_H\frac{\partial\bar{x}}{\partial\bar{p}} &= (\bar{p}f'_H + C''(a_H))\frac{\partial a_H}{\partial\bar{p}} \Leftrightarrow \\ \frac{\partial a_H}{\partial\bar{p}} &= \frac{f_H}{\bar{p}f'_H + C''(a_H)} + \frac{\bar{p}f'_H}{\bar{p}f'_H + C''(a_H)}\frac{\partial\bar{x}}{\partial\bar{p}}. \end{aligned}$$

Implicit differentiation of the consumer indifference condition (11) yields $\partial\bar{x}/\partial\bar{p}$ as follows.

$$-f_H(1-g) + (1-g)(v_H - \bar{p})f'_H\left(\frac{\partial\bar{x}}{\partial\bar{p}} - \frac{\partial a_H}{\partial\bar{p}}\right) = gf_L + g(\bar{p} - v_L)f'_L\left(\frac{\partial\bar{x}}{\partial\bar{p}} - \frac{\partial a_L}{\partial\bar{p}}\right) \Rightarrow$$

$$\frac{\partial\bar{x}}{\partial\bar{p}} = \frac{gf_L + (1-g)f_H}{(1-g)(v_H - \bar{p})f'_H - g(\bar{p} - v_L)f'_L}$$

$$+\frac{(1-g)(v_H - \bar{p})f'_H}{(1-g)(v_H - \bar{p})f'_H - g(\bar{p} - v_L)f'_L}\frac{\partial a_H}{\partial\bar{p}} - \frac{g(\bar{p} - v_L)f'_L}{(1-g)(v_H - \bar{p})f'_H - g(\bar{p} - v_L)f'_L}\frac{\partial a_L}{\partial\bar{p}}.$$

Now, suppose that $\partial\bar{x}/\partial\bar{p} = 0$. Then combining the calculations above, $\partial\bar{x}/\partial\bar{p} = 0$ implies that

$$\frac{gf_L + (1-g)f_H}{(1-g)(v_H - \bar{p})f'_H - g(\bar{p} - v_L)f'_L}$$
$$+\frac{(1-g)(v_H - \bar{p})f'_H}{(1-g)(v_H - \bar{p})f'_H - g(\bar{p} - v_L)f'_L}\frac{f_H}{\bar{p}f'_H + C''(a_H)}$$
$$-\frac{g(\bar{p} - v_L)f'_L}{(1-g)(v_H - \bar{p})f'_H - g(\bar{p} - v_L)f'_L}\frac{f_L}{\bar{p}f'_L + C''(a_L)} = 0,$$

which implies after reorganizing

$$0 = \frac{gf_L}{(1-g)(v_H - \bar{p})f'_H - g(\bar{p} - v_L)f'_L}\left(1 - \frac{\bar{p} - v_L}{\bar{p}}\frac{\bar{p}f'_L}{\bar{p}f'_L + C''(a_L)}\right)$$
$$+\frac{(1-g)f_H}{(1-g)(v_H - \bar{p})f'_H - g(\bar{p} - v_L)f'_L}\left(1 + \frac{v_H - \bar{p}}{\bar{p}}\frac{\bar{p}f'_H}{\bar{p}f'_H + C''(a_H)}\right) \neq 0,$$

which is a contradiction. To see this note that $C''(a_L) > 0$ implies that

$$0 < \bar{p} - v_L\bar{p}\frac{\bar{p}f'_L}{\bar{p}f'_L + C''(a_L)} < 1,$$

and

$$\frac{v_H - \bar{p}}{\bar{p}}\frac{\bar{p}f'_H}{\bar{p}f'_H + C''(a_H)} < -1 \Leftrightarrow$$
$$C''(a_H) < v_H f'_H,$$

which contradicts with Assumption 3.

Now, we established that $\partial\bar{x}/\partial\bar{p}$ cannot be 0. But this implies by Lemma 4 that $\partial\bar{x}/\partial\bar{p}$ is globally positive.

Proof of Proposition 3 We will start with the effective manipulation by L-type. L-type does effective manipulation if and only if $a_L > \bar{x} - \underline{x}$, which is equivalent to $\underline{x} > \bar{x} - a_L$. By Corollary 1 this is equivalent to $\mathbb{E}(v|\bar{x} - a_L) < \bar{p}$ if manipulation is restricted to be 0. That is, when there is no manipulation, a consumer who receives a signal that equals to $\bar{x} - a_L$, should expect that the quality should be lower than the price $\bar{p}$. When we calculate the posterior expectation of such a consumer

$$\frac{(1-g)v_H f(\bar{x}-a_L-v_H)+gv_L f(\bar{x}-a_L-v_L)}{(1-g)f(\bar{x}-a_L-v_H)+gf(\bar{x}-a_L-v_L)} < \bar{p}.$$
$$(1-g)(v_H-\bar{p})f(\bar{x}-a_L-v_H)-g(\bar{p}-v_L)f(\bar{x}-a_L-v_L)<0$$
$$=(1-g)(v_H-\bar{p})f(\bar{x}-a_H-v_H)-g(\bar{p}-v_L)f(\bar{x}-a_L-v_L)\Leftrightarrow$$
$$f(\bar{x}-a_L-v_H)<f(\bar{x}-a_H-v_H)\Leftrightarrow|\bar{x}-a_L-v_H|>|\bar{x}-a_H-v_H|.$$

In sum,

$$a_L>\bar{x}-\underline{x}\Leftrightarrow|\bar{x}-a_L-v_H|>|\bar{x}-a_H-v_H|. \tag{34}$$

We will show below that this equivalence condition for L-type's effective manipulation holds when $\bar{p}<(1-g)v_H+gv_L$ or when $\bar{p}>(1-g)v_H+gv_L$ and $\bar{x}>(a_L+a_H)/2$ $+\,v_H$.

Now, when $\bar{p}<(1-g)v_H+gv_L$, $a_L>a_H$ by Proposition 1. Then by first-order conditions (7)

$$f(\bar{x}-a_L-v_L)>f(\bar{x}-a_H-v_H)\Leftrightarrow|\bar{x}-a_L-v_L|<|\bar{x}-a_H-v_H|,$$

since $f(\cdot)$ is unimodal. Then by Lemma 1, the condition above is implies equivalent to

$$\bar{x}<a_H+v_H,$$
$$\bar{x}-a_H-v_H<\bar{x}-a_L-v_L<a_H+v_H-\bar{x}\Rightarrow$$
$$\bar{x}<\frac{a_L+a_H}{2}+\frac{v_L+v_H}{2}<\frac{a_L+a_H}{2}+v_H$$

On the other hand, when $a_L>a_H$, condition 34 is equivalent to

$$\bar{x}<a_L+v_H$$
$$\bar{x}-a_L-v_H<\bar{x}-a_H-v_H<a_L+v_H-\bar{x}\Leftrightarrow$$
$$\bar{x}<\frac{a_L+a_H}{2}+v_H,$$

which holds when $\bar{p} < (1-g)v_H + gv_L$.

When $\bar{p} > (1-g)v_H + gv_L$, $a_H > a_L$ by Proposition 1. Therefore, condition 34 becomes equivalent to

$$\bar{x} > a_L + v_H$$
$$a_L + v_H - \bar{x} < \bar{x} - a_H - v_H < \bar{x} - a_L - v_H \Leftrightarrow$$
$$\bar{x} > \frac{a_L + a_H}{2} + v_H,$$

because of Lemma 1. This completes the argument for effective manipulation by L-type.

By a similar argument as above, H-type does effective manipulation if and only if

$$a_H > \bar{x} - \underline{x} \Leftrightarrow \underline{x} > \bar{x} - a_H$$
$$(v_H - \bar{p})(1-g)f(\bar{x} - a_H - v_H) - (\bar{p} - v_L)gf(\bar{x} - a_H - v_L) < 0$$
$$= (v_H - \bar{p})(1-g)f(\bar{x} - a_H - v_H) - (\bar{p} - v_L)gf(\bar{x} - a_L - v_L) \Leftrightarrow$$
$$f(\bar{x} - a_L - v_L) < f(\bar{x} - a_H - v_L) \Leftrightarrow |\bar{x} - a_L - v_L| > |\bar{x} - a_H - v_L|.$$

In sum,

$$a_H > \bar{x} - \underline{x} \Leftrightarrow |\bar{x} - a_L - v_L| > |\bar{x} - a_H - v_L|. \tag{35}$$

When $\bar{p} < (1-g)v_H + gv_L$, $a_H < a_L$, therefore condition (35) is equivalent to

$$\bar{x} < a_L + v_L,$$
$$\bar{x} - a_L - v_L < \bar{x} - a_H - v_L < a_L + v_L - \bar{x} \Leftrightarrow$$
$$\bar{x} < v_L + \frac{a_L + a_H}{2}.$$

When $\bar{p} > (1-g)v_H + gv_L$, $a_H > a_L$, therefore condition (35) is equivalent to

$$\bar{x} > a_L + v_L$$

$$a_L + v_L - \bar{x} < \bar{x} - a_H - v_L < \bar{x} - a_L - v_L \Leftrightarrow$$

$$\bar{x} > v_L + \frac{a_L + a_H}{2},$$

which holds as long as $a_H > a_L$.

Proof of Proposition 4 For notational simplicity let $f_j := f(\bar{x} - v_j - a_j)$ and $F_j := F(\bar{x} - v_j - a_j)$ for $j \in \{H, L\}$. In the absence of advertising signal consumer purchases the product if and only if $\bar{p} \leq gv_L + (1-g)v_H$, i.e. whenever expected quality under prior beliefs exceeds the price. Therefore, ex-ante (expected) consumer utility (surplus) without any advertising signal is equal to

$$E_{NS}(CS) = \begin{cases} gv_L + (1-g)v_H, & \text{if} \quad \bar{p} \leq gv_L + (1-g)v_H \\ 0, & \text{if} \quad \text{otherwise.} \end{cases}$$

On the other hand, when the consumer acts upon the biased advertising signal, expected consumer surplus will be

$$E_S(CS) = g(1 - F_L)(v_L - \bar{p}) + (1-g)(1 - F_H)(v_H - \bar{p})$$

We need to show that $E_S(CS) \geq E_{NS}(CS)$ always holds. When consumer decides based on advertising signals, the signal threshold for purchasing decision is given by equation 10 which in turn implies that

$$\frac{(1-g)(v_H - \bar{p})}{g(\bar{p} - v_L)} = \frac{f_L}{f_H}. \tag{36}$$

Consider the first case, i.e, $\bar{p} \le gv_L + (1-g)v_H$. Then,

$$\begin{aligned} E_S(CS) \ge E_{NS}(CS) &\Leftrightarrow \\ -g(v_L - \bar{p})F_L - (1-g)(v_H - \bar{p})F_H &\Leftrightarrow \\ \frac{F_L}{F_H} \ge \frac{(1-g)(v_H - \bar{p})}{g(\bar{p} - v_L)} = \frac{f_L}{f_H} &\Leftrightarrow \quad (37) \\ \frac{f_L}{F_L} \le \frac{f_H}{F_H} & \quad (38) \end{aligned}$$

where the equality in 37 follows from equation 36. By Lemma 1, $a_L + v_L < a_H + v_H$. Therefore, $\bar{x} - v_L - a_L > \bar{x} - v_H - a_H$. Also, since f is log-concave by Assumption 1, F must also be log-concave. Combining these two observations we obtain the inequality in 38 as desired.

Now consider the remaining case that $\bar{p} > gv_L + (1-g)v_H$. Then,

$$\begin{aligned} E_S(CS) \ge E_{NS}(CS) &\Leftrightarrow \\ g(v_L - \bar{p})(1 - F_L) + (1-g)(v_H - \bar{p})(1 - F_H) \ge 0 &\Leftrightarrow \\ \frac{1 - F_L}{1 - F_H} \le \frac{(1-g)(v_H - \bar{p})}{g(\bar{p} - v_L)} = \frac{f_L}{f_H} &\Leftrightarrow \quad (39) \\ \frac{f_L}{1 - F_L} \ge \frac{f_H}{1 - F_H} & \quad (40) \end{aligned}$$

where, as before, the equality in 39 follows from equation 36. Since f is log-concave, $1 - F$ is also log-concave. This in turn implies that $f(x)/(1 - F(x))$ is increasing in x. Since $\bar{x} - v_L - a_L > \bar{x} - v_H - a_H$ always holds, we obtain the inequality in 40 as desired.

Proof of Proposition 5 Consider any strictly positive couple of levels advertisement $a_L, a_H >> 0$. Recall that the consumers' purchasing decisions are determined by the following equation:

$$\frac{v_H f(\bar{x} - v_H - a_H)}{f(\bar{x} - v_H - a_H) + f(\bar{x} - a_L)} = \bar{p}.$$

The left-hand does not change if we subtract $\min\{a_L, a_H\}$ from a_L, a_H and $\bar{x}$ since

$$\frac{v_H f(\bar{x} - \min\{a_L, a_H\} - v_H - (a_H - \min\{a_L, a_H\}))}{\sum_j f(\bar{x} - \min\{a_L, a_H\} - v_j - (a_j - \min\{a_L, a_H\}))}$$
$$= \frac{v_H f(\bar{x} - v_H - a_H)}{f(\bar{x} - v_H - a_H) + f(\bar{x} - a_L)} = \bar{p}.$$

That is, if the monopoly reduces both advertisement levels by $\min\{a_L,\ a_H\}$, the consumers' unique response will be to reduce $\bar{x}$ by $\min\{a_L,\ a_H\}$, since they expect the monopoly to do manipulation at a uniformly lower level.

Since manipulation is costly, the monopoly can always achieve a higher profit by reducing the minimum advertisement level to 0. ■

Proof of Proposition 6 Let $((p_L, a_L), (p_H, a_H))$ be any pure strategy separating equilibrium. By definition $p_L \neq p_H$ and suppose for a moment both prices are strictly less than v_H. Then all consumers would buy the high type at price p_H, which gives incentive to L-type to imitate H type $p_H > 0$. On the other hand, no equilibrium with $p_H \in \{0, v_H\}$ would be strict since either the monopoly or the consumers would be indifferent with other strategies.

Proof of Theorem 7 When $v_L = 0$, the description of advertising strategies and their existence is same as in the exogenous price case, as described by Theorem 1. The prices can be supported by the off-equilibrium beliefs of the consumers that any deviation to another price means that the monopolist is of L-type.

When $v_L > 0$, now the monopolist has the outside option of choosing v_L and making positive profit. Therefore, for extremely high prices, where the expected sales are very close to zero, because $\bar{x} \to \infty$, the monopolist may choose to deviate to v_L.

Proof of Theorem 2 The proof consists of two stages. In the first stage, we will argue that equations (23) to (26) describe a partial separating equilibrium. Then in the second stage, we will prove that the equations (23) to (26) has a solution for high enough prices.

For L type to mix between two prices, it should be indifferent between the profit levels at the two prices. When L-type charges v_L all consumers buy the good since their expected consumer surplus is at least 0. Therefore, the Monopoly will sell the product to the whole market providing the profit of v_L to the Monopoly. If the L type chooses the price $\bar{p}$ with the corresponding advertising level a_L the expected profit would be given as the RHS of the equation (23). The equation (23) ensures that L-type is indifferent between choosing v_L and $\bar{p}$.

When consumers observe $\bar{p}$ they remain uncertain about the type of the monopoly because both types could have chosen this price level. Therefore, all consumers consult to the private signals they receive. Suppose for a moment that consumers use a threshold strategy $\bar{x}$ for their purchasing decision when they observe the price $\bar{p}$. Expecting that consumers use this threshold, the optimal choice of manipulation levels each type are given by the first order conditions (24) and (25).

Given the manipulation levels a_L and a_H, consumers infer that the price $\bar{p}$ could be chosen by the L type with probability $\bar{\alpha}/(1+\bar{\alpha})$ and H-type with probability $1/(1+\bar{\alpha})$. Then given the private signal x_i, the expected quality level is

$$\frac{v_H f(\bar{x} - v_H - a_H) + v_L \bar{\alpha} f(\bar{x} - v_L - a_L)}{f(\bar{x} - v_H - a_H) + \bar{\alpha} f(\bar{x} - v_L - a_L)}.$$

Consumers are indifferent when they receive the threshold signal $\bar{x}$. After some re-arrangement, the indifference condition can be written as equation (26).

To establish the existence, first note that for every value of $\bar{x}$ and $\bar{p}$ the manipulation levels a_L and a_H are bounded since the cost function is unbounded but the pdf function $f(\cdot)$ is bounded. Therefore, by equation (23) $\bar{x}$ should be finite as well. Now, as $\bar{x} \to \infty$ RHS of equation (23) converges to $-C(a_L) < 0$. As $\bar{x} \to -\infty$ the RHS of equation (23) converges to $\bar{p} - C(a_L)$. When we substitute a_L from equation (24), RHS of equation (23) becomes

$$\bar{p}(1 - F(\bar{x} - a_L - v_L)) - C((C')^{-1}(\bar{p} f(\bar{x} - a_L - v_L))),$$

which converges to $\bar{p} > v_L$.

Now, we need to check two inequalities; $\bar{\alpha} < 1$ and the profit of the H-type being greater than v_L. The latter follows from the quality advantage of the H type. Even if the H type chooses the same manipulation level as the L-type, its profit will be

$$\bar{p}(1 - F(\bar{x} - a_L - v_H)) - C(a_L) > \bar{p}(1 - F(\bar{x} - a_L - v_L)) - C(a_L) = v_L.$$

For $\bar{\alpha}$, consider equation (26) and (23). As $\bar{p}$ converges to v_H, (23) implies that $\bar{x}$ converges to a finite value. Given that, equation (26) implies that $\bar{\alpha}$ converges to 0. Therefore, there exists $\tilde{p} < v_H$ such that for all $\bar{p} > \tilde{p}$ $\bar{\alpha} < 1$. This establishes the existence.

Proof of Proposition 8 Implicitly differentiating the incentive compatibility condition for L-type, equation (23) implies

$$1 - F(\bar{x} - a_L - v_L) - \bar{p}f(\bar{x} - a_L - v_L)\left(\frac{\partial \bar{x}}{\partial \bar{p}} - \frac{\partial a_L}{\partial \bar{p}}\right) - C'(a_L)\frac{\partial a_L}{\partial \bar{p}} = 0.$$

Substituting the first-order condition for L-type, equation (24) acquires

$$\frac{\partial \bar{x}}{\partial \bar{p}} = \frac{1 - F(\bar{x} - a_L - v_L)}{\bar{p}f(\bar{x} - a_L - v_L)} > 0. \tag{41}$$

The behavior of a_L and π_H depends on the monotonicity of hazard rate of the noise distribution. To show the monotonicity note that the c.d.f. F being log-concave implies that the ratio f/F is a decreasing function. Moreover, the derivative of f/F is negative, that is, $f'F - f^2 < 0 \Leftrightarrow f^2 > f'F$. Since the noise distribution is symmetric, this also implies that $f^2 > (-f')(1-F)$. Moreover, the ratio $(1-F)/f$ is also decreasing.

Implicitly differentiating the first-order condition for L-type, equation (24) yields

$$\frac{\partial a_L}{\partial \bar{p}} = \frac{f^2(\bar{x} - a_L - v_L) + f'(\bar{x} - a_L - v_L)(1 - F(\bar{x} - a_L - v_L))}{f(\bar{x} - a_L - v_L)(\bar{p}f'(\bar{x} - a_L - v_L) + C''(a_L))} > 0,$$

since F is log-concave, which implies that nominator is positive, and second-order condition for the manipulation decision of L-type implies that the denominator is positive.

Finally, the implicit derivative of the profit function after substituting the first-order condition for H-type, equation (25)

$$\frac{\partial \pi_H}{\partial \bar{p}} = 1 - F(\bar{x} - a_H - v_H) - \bar{p}f(\bar{x} - a_H - v_H)\frac{\partial \bar{x}}{\partial \bar{p}} > 0 \Leftrightarrow$$
$$\frac{1 - F(\bar{x} - a_H - v_H)}{f(\bar{x} - a_H - v_H)} > \frac{1 - F(\bar{x} - a_L - v_L)}{f(\bar{x} - a_L - v_L)},$$

which always holds by Assumption 4.

Proof of Proposition 9 Equation (26) implies that when $\bar{p} = \hat{p}_{\bar{\alpha}}$,

$$f(\bar{x} - a_L - v_L) = f(\bar{x} - a_H - v_H),$$

which implies that $a_L = a_H$ by the first-order conditions (24) and (25). Moreover by a comparison, $a_L > a_H$ if and only if $\bar{p} < \hat{p}_{\bar{\alpha}}$.

To show that L-type always effectively manipulates, compare the indifference conditions (23) and (27). Since

$$\bar{p}(1 - F(\underline{x} - v_L)) = v_L = \bar{p}(1 - F(\bar{x} - a_L - v_L)) - C(a_L),$$

the sales of L-type when there is manipulation is always greater than its sales when there is no manipulation; that is, $a_L > \bar{x} - \underline{x}$. Then, since when $\bar{p} \geq \hat{p}_{\bar{\alpha}}$, $a_H \geq a_L$, we have

$$a_H \geq a_L > \bar{x} - \underline{x},$$

which completes the proof. ■

Proof of Lemma 3 The rate of change of profit of type j among the Pooling equilibria is given by

$$\frac{\partial \pi_j}{\partial p} = F(a_j + v_j - \bar{x}) - p\frac{\partial \bar{x}}{\partial p} f(a_j + v_j - \bar{x}),$$

which is positive if and only if

$$\frac{\partial \bar{x}}{\partial p} p < \frac{F(a_j + v_j - \bar{x})}{f(a_j + v_j - \bar{x})}.$$

Since, $a_H + v_H > a_L + v_L$ and F/f is increasing, since F is log-concave, if $\partial \pi_L / \partial p$ > 0, then so is $\partial \pi_H / \partial p$. Then Assumption 5 implies that $p_H > p_L$, since

$$\frac{\partial \bar{x}}{\partial p} p_L = \frac{F(a_L + v_L - \bar{x})}{f(a_L + v_L - \bar{x})} < \frac{F(a_H + v_H - \bar{x})}{f(a_H + v_H - \bar{x})}.$$

■

Proof of Proposition 11 The sufficiency of expectations specified in equation (30) is straightforward. When the monopoly of either type deviates to any other price $p \in$

(v_L, v_H) from the price $\bar{p}$ prescribed by the equilibrium, it faces zero demand because consumers believe with certainty that the deviating type is v_L.

The necessity of expectations specified in equation (31) comes from Lemma 3. For any price $\bar{p} \in (v_L, v_H)$, if the deviation faces with the demand associated with a belief at least as favorable as the prior beliefs, at least one of the types would deviate to p_L or p_H to achieve a higher profit. Since $p_H \neq p_L$, there is no price that prior beliefs as "pre-signal" off-equilibrium beliefs could support a pooling equilibrium at that price. ■

Proof of Proposition 12 We first show that any pooling equilibria that supports a price that is outside of the interval $[\bar{p}_L, \bar{p}_H]$ is defeated by another pooling equilibrium. This argument has two parts and the second part is very similar to the first part, so we will present only the first part.

Let $\bar{p} < \bar{p}_L$ and consider any two pooling equilibria that support $\bar{p}$ and $\bar{p}_L$ respectively. The set of types, K that might want to deviate from $\bar{p}$ to $\bar{p}_L$ is the both types. That is, $K = \{L, H\}$, and for each of the types the equilibrium payoff at $\bar{p}_L$ is strictly higher than $\bar{p}$. To see that condition 3 in the definition 1 also holds, note that for $\bar{p}$ to be supported by a pooling equilibrium, the consumers' off-equilibrium pre-signal belief $P_{\bar{p}}(v_H|\bar{p}_L) < P(v_H)$. Therefore, consumers' off equilibrium belief for any deviation from $\bar{p}$ to $\bar{p}_L$ cannot be the same as the Bayesian update prescribed in condition 3 in definition 1.

A similar argument shows that any pooling equilibrium that supports $\bar{p} > \bar{p}_H$ is defeated by a pooling equilibrium that supports $\bar{p}_H$.

It is clear that two equilibria that differ only with respect to their off-equilibrium beliefs cannot defeat each other. Moreover, no two pooling equilibria that support two different prices in $[\bar{p}_L, \bar{p}_H]$ can defeat each other, since by Assumption 5 one of them offers a higher profit to one of the types. ■

Proof of Proposition 13 Consider any two partially separating equilibria indexed by the prices $p_1 < p_2$. Suppose that the first equilibrium assigns $P_1(v_H|p') < 0.5$ to any off-equilibrium price $p' \neq p_1$. Our claim is that the partially separating equilibrium that supports p_2 defeats the first equilibrium.

First, note that p_2 is not in the prescription first equilibrium for both types. Therefore $K = \{L, H\}$. The profit of L does not change from first equilibrium to the second as it is fixed at v_L. On the other hand, the profit for H-type is strictly higher in the second equilibrium by Proposition 8. Examining condition 3 in the definition 1 reveals that consumers' possible off-equilibrium beliefs after observing a deviation from p_1 to p_2 assign a probability to the H-type that ranges from 0.5 to 1. This probability is higher than the prescribed off-equilibrium beliefs to the first equilibrium by hypothesis. ■

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